A Probability Theory Based Price Determination Framework for Utility Companies in an Oligopolistic Energy Market

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Abstract—**Distributed power generation and distribution network with the dynamic pricing scheme are the major trend of the future smart grid. A smart grid is a network which contains multiple non-cooperative utility companies that offer time-of-use dependent energy prices to energy consumers and aim to maximize their own profits. Decentralized power network allows each energy consumer to have multiple choices among different utility companies. In this paper, an optimization framework is introduced to determine the energy price for utility companies in an oligopolistic energy market. At the beginning of each billing period (a day), each utility company will announce the time-of-use dependent pricing policy during the billing period, and each energy consumer will subsequently choose a utility company for energy supply to minimize the expected energy cost. The energy pricing competition among utility companies forms an *n*-person game because the pricing strategy of each utility company will affect the profits of others. To be realistic, the prediction error of a user's energy consumption is properly accounted for in this paper and is assumed to satisfy certain probability distribution at each time slot. We start from the most commonly-used normal distribution and extend our optimization framework to a more general case. A Nash equilibrium-based pricing policy is presented for the utility companies and the uniqueness of Nash equilibrium is proved. Experimental results show the effectiveness of our game theoretic price determination framework.**

Index Terms—**smart grid; dynamic pricing; probability theory; oligopolistic market.**

I. INTRODUCTION

The current smart grid technology is undergoing a transformation from the traditional static and centralized structure to one that is more distributed and consumer-interactive [1][2]. One of the major modifications is the increasingly widespread application of distributed generation, which is the integration of small-scale power generation facilities located close to the load devices [1]. Together with the lowered costs, improved reliability, reduced emissions and expanded energy options, the introduction of decentralized electrical network architecture also makes it more challenging to match power supply to real-time demand for the power generation and distribution networks [3]. It is generally recognized that the peak demand rather than the average determines the amount of generation, transmission and distribution capacities that utility companies need to provision. The huge difference between energy consumption levels at peak usage time and off-peak hours has resulted in not only cost inefficiencies and potential electricity supply failures (brownouts and blackouts), but also environmental pollution due to the over-provisioning of the power grid and the resulting energy waste [4]. For example, the US national load factor (i.e. average load divided by the peak load) is about 55%, and only 10% of generation plants and 25% of distribution facilities are used more than 400 hours per year, i.e., 5% of the total time [1].

Dynamic energy pricing gives a potential solution for demand shaping to reduce the peak power demand and smoothen the overall power profile, and has been widely investigated in the past decade [1]-[7]. Authors in [4] and [5] provide algorithms for optimal load scheduling and cost minimization incentivized by dynamic energy pricing, whereas reference [6] provides a real-time pricing policy for the utility companies with the assumption that each energy consumer will optimize a predefined utility function. Considering the fact that utility companies tend to make decisions based on the anticipated response from the users, a concurrent optimization model for both the utility company and energy users is presented in [7] in order to optimize the social welfare. The optimization framework in [7] is based on a centralized monopolistic electrical grid, where a single utility company supplies all the power demands for the electricity consumers in a local area. However, as the decentralized smart grid is the major trend of the future electrical power network architecture [1], each homeowner is allowed to have multiple choices among different utility companies. Hence, the competition between different utility companies should be accounted for. Based on this observation, authors in [12] tackle the profit maximization problem of non-cooperative utility companies in an oligopolistic electricity market, and provide a Nash equilibrium-based optimal pricing policy for the utility companies.

However, there are some problems with the previous work. Almost all assume that the energy consumption for each homeowner is accurately predictable and is given in prior to utility companies, which is far from realistic.

Although the historical data can be used to help utility companies to make predictions, accurate prediction of load power demand is still hard or even impossible since the power demand depends on exogenous factors and varies dramatically as a function of time of day and seasonal factors [7]. According to the algorithms presented in the above papers, a forecasting error of energy consumption may lead to a completely different solution. For example, the previous paper [12] assumes that each homeowner will choose one utility company with a certain probability according to the pricing policies, and the objective function for the utility companies is to maximize the predicted total profit. This kind of strategy will lead to a huge uncertainty because the variance of the total profit due to prediction error is never accounted for.

In this paper, we propose an optimization framework of dynamic pricing for utility companies to maximize their profits in an oligopolistic energy market. At the beginning of each billing period (i.e., a day), each utility company will announce the time-of-use dependent pricing policy during the billing period, and each energy consumer will subsequently choose a utility company for energy supply to minimize the expected energy cost. Prediction of the energy demand of each user is required (at both the user side and the utility company side) at the beginning of each billing period, and we properly account for the prediction error of user's energy consumption in this paper by assuming that it satisfies certain probability distribution at each time slot. We start from the most commonly-used normal distribution and then extend to more general distributions. Instead of simply maximizing the expected profit, each utility company optimizes a payoff function that is a combination of maximizing expectation and minimizing the variance of the total profit, accounting for the prediction error. The energy pricing competition among utility companies forms an *n*-player game because the pricing strategy of each utility company will affect the profits of others. We present a Nash equilibrium-based optimal solution for the utility companies to determine the time-of-use pricing policy and prove the uniqueness of the Nash equilibrium point.

The remainder of this paper is organized as follows. In the next section, we present the system model as well as the optimal pricing policy for utility companies under the assumption that the prediction error obeys a normal distribution at each time slot. The optimization framework is then extended to a more general case in Section III. Section IV provides the simulation results, and the paper is concluded in Section V.

II. System Model under the Normal Distribution Assumption

In this paper, a slotted time model is assumed for all models, i.e., all system cost parameters and constraints as well as scheduling decisions are provided for discrete time intervals of constant length. The scheduling epoch is thus divided into a fixed number of equal-sized time slots (in the experiment, a day is divided into 24 time slots, each with duration of one hour). A unified electricity price is used throughout the paper.

We define *Price function,* $P_i[t]$*,* as the price of one unit of energy (kWh) for each utility company i at time slot t. The price is decided by the utility company and pre-announced to homeowners. In addition, for every homeowner j, $con_j[t]$ is the total energy consumption at time t. It can be easily observed that the equation below calculates the total energy cost for a certain homeowner j if he chooses company i:

$$cost_{i,j} = \sum_t P_i[t] \cdot con_j[t]$$

Previous papers assumed that $con_j[t]$ is an accurately predictable value and is given based on the prediction from previous data, which turns out to be oversimplified because of the following two reasons: First of all, there are many random disturbances and uncertainties in real power systems, which may lead to prediction error [14]; Further more, the previous data of energy consumption itself can vary dramatically for external factors as well as homeowners' own habits. As a result, what we can do is to predict the distribution of energy consumption from the previous observations.

According to these factors, a probabilistic model is presented in this paper, which means during the prediction step, $con_j[t]$ is considered as a stochastic behavior and obeys a certain distribution. We start from the most popular distribution: normal distribution whose *probability density function* (PDF) is defined by the formular

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where μ and σ^2 are the mean and variance of the random variable. For energy consumption $con_j[t]$, we use *expection* $\mathbf{E}[con_j[t]]$ and a *variance* $\mathbf{Var}[con_j[t]]$ to represent these 2 parameters. The former parameter stands for the average predicted energy consumption while the latter one tells how far the energy consumption may be spread out, i.e., the prediction error. In this paper, we assume $con_j[t]$ are independent random variables for different j and t.

Property 1: $cost_{i,j}$ obeys a normal distribution with the expectation and variance calculated as follows:

$$\mathbf{E}[cost_{i,j}] = \sum_t P_i[t] \cdot \mathbf{E}[con_j[t]]$$

$$\mathbf{Var}[cost_{i,j}] = \sum_t (P_i[t])^2 \cdot \mathbf{Var}[con_j[t]]$$

Proof: as $P_i[t]$ values are given, $cost_{i,j}$ is a linear function of $con_j[t]$. As $con_j[t]$ are predicted to be normal distribution variables and independent from each other, based on the properties of normal distribution in [15], $con_j[t]$ will also obey a normal distribution. ∎

The distributed power network offers more options for homeowners to select one utility company among different options. Previous papers presented two kinds of models to determine the probability. In [10], each homeowner has a "threshold price" and will equally choose a company which offers a price lower than threshold. This model is unrealistic because it is a common sense that buyers should have a higher probability to choose a seller who offers a cheaper price. And also, the model fails to take into account the interactions among different utility companies. In this paper, we use a modified model based on the demand function in [12] and use the expectation of total energy cost to determine the probability that each utility company i is chosen by a certain homeowner j ($prob_{i,j}$)

$$prob_{i,j} = \frac{e^{-\mathbf{E}[cost_{i,j}]}}{e^{-\mathbf{E}[cost_{i,j}]} + \sum_{k\neq i} e^{-\mathbf{E}[cost_{k,j}]}}$$

which reveals that even considering the information asymmetry, energy consumers will still have a preference to choose the company who offers a cheaper price.

To study the expectation and variance of the profit from a certain homeowner, we use some abbreviations in the following equations: p is used to represent the probability $prob_{i,j}$, which can be observed to be deterministic and is a function of $P_i[t]$. μ and σ^2 are used to represent the mean and variance of the $cost_{i,j}$. And also we use a function $N(x)$ to represent the PDF of $cost_{i,j}$, which is proved to be normal distributed. There are then two outcomes for each homeowner as follows:

1. With a probability of p, the homeowner will choose this certain company. The profit of the company is the same as the cost function of the homeowner, which obeys a normal distribution.
2. With a probability of 1-p, the homeowner will not choose this certain company. The profit of the company from this homeowner is 0.

Considering these two outcomes, the expectation and variance of the profit from the certain homeowner can be calculated using conditional probability formulation as follows:

$$\begin{aligned}\mathbf{E}[profit_{i,j}] &= p\cdot \mathbf{E}[profit_{i,j}|\textit{the company is chosen}] + \\ &\quad (1-p)\cdot \boldsymbol{E}[profit_{i,j}|\textit{the company is not chosen}] \\ &= p\mu\end{aligned}$$

$$\begin{aligned}\mathbf{Var}[profit_{i,j}] &= p\cdot \mathbf{Var}[profit_{i,j}|\textit{the company is chosen}] + \\ &\quad (1-p)\cdot \mathbf{Var}[profit_{i,j}|\textit{the company is not chosen}] \\ &= p\int N(x)(x-\mathbf{E}[profit_{i,j}])^2dx + (1-p)(0-\mathbf{E}[profit_{i,j}])^2 \\ &= p\int \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}(x-p\mu)^2dx + (1-p)(p\mu)^2 \\ &= p\int \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}(x-\mu+u-p\mu)^2dx + (1-p)(p\mu)^2 \\ &= p\int \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}(x-\mu)^2dx \\ &\quad + p(1-p)\mu\int \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}(x-\mu)dx \\ &\quad + p(1-p)^2\mu^2\int \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx + (1-p)(p\mu)^2\end{aligned}$$

By definition of normal distribution, we can observe:

$$\int \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}(x-\mu)^2dx = \sigma^2$$

$$\int \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}(x-\mu)dx = \int \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}(x-\mu)d(x-\mu) = 0$$

$$\int \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx = 1$$

and then we can determine

$$\begin{aligned}\mathbf{Var}[profit_{i,j}] &= p\sigma^2 + p(1-p)^2\mu^2 + (1-p)(p\mu)^2 \\ &= p\sigma^2 + p(1-p)\mu^2\end{aligned}$$

In the power system where the energy consumption is not accurately predictable, utility companies always need to make a tradeoff between the expected profit and the potential risk in profit. Based on the study in [13], a *utility function* $U_{i,j}$ is used to weigh the risk and gains for utility companies:

$$U_{i,j} = \mathbf{E}[profit_{i,j}] - \beta_i\cdot \mathbf{Var}[profit_{i,j}]$$

which means utility function has a positive relation with the profit expectation and a negative relation with the profit variance. In this equation, β_i is a parameter that tells the company's *risk aversion coefficient*. The higher this value, the larger amount of expected profit the company is willing to sacrifice in order to avoid potential risk.

Then the price determination problem for utility companies can be defined as follows:

For each company $\boldsymbol{i}$, **find** $P_i[t]$ that

$$\textbf{max}\quad utility\_total_i = \sum_j U_{i,j}$$

$$\textbf{s.t.}\quad P_i[t] > 0,\quad \forall i,t$$
$$P_i[t] \le max\_price,\quad \forall i,t$$

As is mentioned in [12], companies are considered as non-cooperative among each other, and the price function of one company will affect the profit of others, which means these companies form an n-person game. Considering this, we are interested in the existence and uniqueness of Nash equilibrium points.

Property 2: Utility companies have a *unique Nash equilibrium point* in the profit maximization problem.

Proof: It has been proven in [12] that if the energy consumption is assumed to be accurately predictable, $profit_{i,j}$ is a *strictly concave* function in terms of energy price functions under a certain modification. This means $\mathbf{E}[profit_{i,j}]$ is a *strictly concave* function in our paper. In reality as well as our own experimental statistics, β_i should be a very small value (usually from 0 to 0.01) because expected profit should always be the major concern in making decisions, so $\beta_i\cdot \mathbf{Var}[profit_{i,j}]$ has very little effect on the convexity of the function. Then $U_{i,j}$ is a *strictly concave* function in our area of interest. Therefore, the utility maximization problem is a strictly concave n-person game. In this case, the existence and uniqueness of Nash equilibrium are directly resulted from the first and third theorem in [9]. ■

The Nash equilibrium point can be calculated using iterative local utility maximization of each utility company based on game theoretic method. In this iterative solution, a constant value d is needed to determine the endpoint. More precisely, the optimization process stopped when no utility companies can achieve a utility function increase higher than d. We can observe that the value of d makes a tradeoff between accuracy of the solution and run-time. According to the uniqueness of the Nash equilibrium point, our solution will be the same under any initial condition (i.e., the initial price function of utility companies). The detailed algorithm is presented as follows:

Algorithm 1: Iterative solution for utility maximization.

Initialize $P_i[t]$ for every i and t.
While (*max_utility_increase* > d)
  For each utility company i:
    Calculate current $utility\_total_i$.
    For each time slot t, set:
      $$\frac{\partial\, utility\_total_i}{\partial P_i[t]} = 0$$
    Solve the equation set to get $P_i[t]$ for each t under the given price functions of other companies.
    Calculate optimized $utility\_total_i$.
  Calculate the *max_utility_increase* among all companies.
$P_i[t]$ for every i and t is determined after the above process.

This problem can be simplified when all companies have the same *risk aversion coefficient.*

Property 3: At the Nash equilibrium point, each utility company has exactly the same price function if β_i stays the same for all companies.

Proof: When β_i is the same for all companies, the objective functions between any two companies are totally the same. Assume the price function of two utility companies, namely c_1 and c_2, are different in a Nash equilibrium point (say $P_{c1}[t] \neq P_{c2}[t]$), another Nash equilibrium point should exist if they exchange their price functions, which contradicts the uniqueness of Nash equilibrium in property 2. In fact, it is always the situation that companies will ultimately use almost the same *risk aversion coefficient* in oligopolistic energy market, due to the interactions and the price competitions among them. ■

The third property offers us an alternative and easier algorithm to solve this problem under this certain situation, as we can simply assume the price function of all utility companies are the same after the first derivative of the objective function. No initial price function is needed and no iteration is required, which makes the algorithm much faster than the previous one. All the price functions can be determined by solving a single equation set. The detailed algorithm is presented as follows:

Algorithm 2: Unique Nash equilibrium point determination.

For any one of utility company i:
  For each time slot t, set:
    $$\frac{\partial\, utility\_total_i}{\partial P_i[t]}\Big|_{P_k[t]=P_i[t]\, for\ all\ k\neq i} = 0$$
  Solve the equation set to get $P_i[t]$ for each t.
For all utility companies $k\neq i$, set price function $P_k[t] = P_i[t]$ for each t.

III. MODEL EXTENSION

In section II, the normal distribution based system model is investigated. However, if we remove this assumption and extend the system model to a more general case, an interesting observation is that all the above solutions still stay the same.

Assume energy consumption is predicted to satisfy a distribution which can be any kind (i.e. normal distribution, uniform distribution or other kind). As $cost_{i,j}$ is a linear function of $con_j[t]$, although we cannot determine the distribution type of $cost_{i,j}$, as long as $con_j[t]$ are independent variables, based on the properties of expectation and variance in [15], we still have

$$\mathbf{E}[cost_{i,j}] = \sum_t P_i[t] \cdot \mathbf{E}[con_j[t]]$$
$$\mathbf{Var}[cost_{i,j}] = \sum_t (P_i[t])^2 \cdot \mathbf{Var}[con_j[t]]$$

Again p is used to represent the probability $prob_{i,j}$, μ and σ^2 are used to represent the mean and variance of the $cost_{i,j}$. This time we use a general function $f(x)$ to represent the PDF of $cost_{i,j}$. By definition of expetation and variance, we have

$$\sigma^2 = \int f(x)(x-\mu)^2 dx$$
$$\mu = \int x f(x) dx$$

and also by definition of probability density function, we have

$$\int f(x) dx = 1$$

Then the expectation and variance of the profit from the certain homeowner can be calculated using conditional probability formulation as follows:

$$\mathbf{E}[profit_{i,j}] = p \cdot \mathbf{E}[profit_{i,j}|the\ company\ is\ chosen] + (1-p)\cdot \boldsymbol{E}[profit_{i,j}|the\ company\ is\ not\ chosen] = p\mu$$

$$\mathbf{Var}[profit_{i,j}] = p \cdot \mathbf{Var}[profit_{i,j}|the\ company\ is\ chosen] + (1-p)\cdot \mathbf{Var}[profit_{i,j}|the\ company\ is\ chosen]$$
$$= p\int f(x)(x-\mathbf{E}[profit_{i,j}])^2 dx + (1-p)(0-\mathbf{E}[profit_{i,j}])^2$$
$$= p\int f(x)(x-p\mu)^2 dx + (1-p)(p\mu)^2$$
$$= p\int f(x)(x-\mu+u-p\mu)^2 dx + (1-p)(p\mu)^2$$

$$= p\int f(x)(x-\mu)^2dx + p(1-p)\mu\int f(x)(x-\mu)dx + p(1-p)^2\mu^2\int f(x)dx + (1-p)(p\mu)^2$$
$$= p\sigma^2 + p(1-p)^2\mu^2 + (1-p)(p\mu)^2$$
$$= p\sigma^2 + p(1-p)\mu^2$$

which is exactly the same as for normal distribution. As a result, all the solutions in section II still stay for any type of distribution in energy consumption.

Although we cannot tell the distribution type of the final profit, a reasonable estimation can be made when the number of homeowners is large enough.

Property 4: If the number of homeowners is large, the total profit of each company will obey an almost normal distribution.

Proof: Energy consumption at each time slot for each homeowner is independent, while the final profit is a linear function of the energy consumption given the designed price functions. Based on the *central limit theorem* presented in [15][1], the mean of a sufficiently large number of independent random variables, each with a well-defined mean and well-defined variance, will be approximately normally distributed. A simple extension of this theorem can be applied here to determine that total profit of each company will finally be approximately normally distributed with the increase of the number of homeowners. ∎

Property 4 is useful for utility companies to make decisions under a certain constraint on potential risk.

IV. Experimental Results

To demonstrate the effectiveness of the proposed solutions, cases corresponding to the aforesaid pricing models are examined. In these simulations, the duration of a time slot is set to one hour. For this reason, power consumption of the tasks is determined with a granularity of one hour. The proposed algorithms have been implemented using Matlab and tested for random cases. During the simulation, the energy consumption at each time slot is predicted to obey a random type of distribution with a well-defined mean and well-defined variance for every homeowner, and each homeowner has a probability to choose a certain utility company based on the offered price functions.

We assume the *risk aversion coefficient* is the same for each utility company and calculated the unique Nash equilibrium point for an oligopolistic energy market containing 5 utility companies serving 100 homeowners. Based on the previous proof, this problem can be easily solved using the second algorithm presented in this paper. However, in order to verify the uniqueness of Nash equilibrium point, we also apply the iterative solution for each utility company under different initial conditions (i.e. initial price functions). As the final calculated unified price function value is between 0.2 and 0.5, we give a unified price of 0.1 as a low initial price and a unified price of 0.9 as a high initial price. To study the effect of *risk aversion coefficient*, we also tested our model under different values of β. Utility function, expected total profit and total profit variance are compared for different cases. The result is presented in Table 1.

Table 1. Utility Function, Profit Expectation and Profit Variance Comparison for the Presented Algorithms under Different Initial Conditions and Different Risk Aversion Coefficients

β	Algorithm and initial price	Utility function	Exp.	Var.
0.000	Algorithm 1, p=0.1	24.99	24.99	25.03
	Algorithm 1, p=0.9	24.99	24.99	25.03
	Algorithm 2	24.99	24.99	25.03
0.002	Algorithm 1, p=0.1	24.88	24.93	24.89
	Algorithm 1, p=0.9	24.88	24.93	24.89
	Algorithm 2	24.88	24.93	24.89
0.004	Algorithm 1, p=0.1	24.77	24.87	24.77
	Algorithm 1, p=0.9	24.77	24.87	24.77
	Algorithm 2	24.77	24.87	24.77
0.006	Algorithm 1, p=0.1	24.66	24.81	24.65
	Algorithm 1, p=0.9	24.66	24.81	24.65
	Algorithm 2	24.66	24.81	24.65
0.008	Algorithm 1, p=0.1	24.54	24.74	24.53
	Algorithm 1, p=0.9	24.54	24.74	24.53
	Algorithm 2	24.54	24.74	24.53
0.010	Algorithm 1, p=0.1	24.44	24.68	24.41
	Algorithm 1, p=0.9	24.44	24.68	24.41
	Algorithm 2	24.44	24.68	24.41

The above table verifies that regardless of how high or low the initial price is set to, utility companies will finally converge to a price function that maximizes their utility functions, and the result is the same for both algorithms. The final price strategy for different companies turned out to be the same as the calculated result, which has been proved to be the unique Nash equilibrium solution.

We also observe that the value β makes a tradeoff between the expected profit and the potential risk in profit. Both the profit expectation and the variance will decrease with the increase of parameter β. The relationship of profit expectation and variance in terms of β is shown in Figure 1.

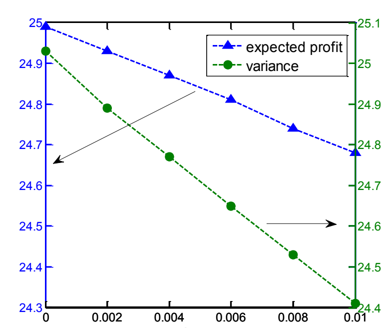

Figure 1. Relationship of Profit Expectation and Variance in Terms of β

To study the exact distribution of the final profit, we randomly generate the energy consumption values at each time slot and calculate the real profit of each utility company, under the well-defined price function based on the result in the previous step. We repeat this experiment for 10000 times and draw the profit distribution graph, as is shown in Figure 2.

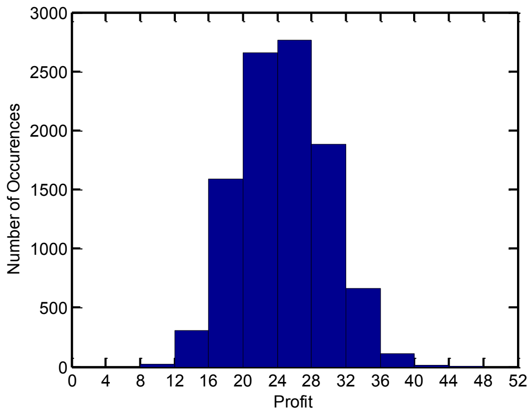

Figure 2. Final Profit Distribution under $\beta = 0.01$

The above figure verifies that regardless of what type of distribution the energy consumption is predicted to obey, the final profit is still almost normally distributed with the calculated mean and variance. This is the same as the predicted result from *central limit theorem*.

The runtime of the proposed algorithms for all the 5 utility companies is about 5 minutes for algorithm 1 and less than 1 second for algorithm 2 respectively both on a machine with a dual core processor with frequency of 2.80 GHz. This run time makes it feasible to utilize our models real-time. The total run-time to test the distribution of final profit under 10000 test cases is around 5 minutes.

V. CONCLUSION

A probability theory based model is presented to tackle the utility maximization problem of non-cooperative companies in oligopolistic energy market considering the prediction error of users' energy consumption. The model is started from the most commonly-used normal distribution and extended to a more general case. An iterative solution is presented and a more efficient algorithm is provided for certain conditions. The model is implemented and tested with some arbitrary test schemes. The results confirm that our designed algorithms lead to Nash equilibrium solutions and also show the effect of risk aversion coefficient on final profit expectation and profit variance. The distribution of final profit satisfies the predicted normal distribution with a calculated mean and variance. The real-time simulation strengthened the reliability of our proposed solution on price function with an acceptable runtime.

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