An Electricity Trade Model for Microgrid Communities in Smart Grid

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***Abstract*—Distributed microgrid network is the major trend of future smart grid, which contains various kinds of renewable power generation centers and a small group of energy users. In the distributed power system, each microgrid acts as a "prosumer" (producer and consumer) and maximizes its own social welfare. In addition, different microgrids can interact among each other through trading over a marketplace. In this paper, two models are introduced for microgrids to deal with the welfare maximization problems. In the first model, a microgrid is considered as a closed economy group and decides the optimal power generation distribution in terms of time. In the second model, each microgrid can trade with its neighborhoods and thus achieve a welfare increase from making use of its comparative advantage on power generation during a certain period of time. For each model, an efficient solution is presented. Experimental result shows the accuracy and efficiency of our presented solutions.**

I. Introduction

The current smart grid technology is undergoing a transformation from a centralized, producer-controlled network to one that is more distributed and consumer-interactive [1][2]. With the introduction of the decentralized network architecture, the entire business model will be changed together with the relationship of all stakeholders, involving and affecting utilities, regulators, energy service providers, technology and automation vendors and all consumers of electric power [2]. Among all the changes in smart grid, the transformation of the power generation network is of the most critical, i.e., instead of being generated by a few far-off high-capacity generators and transmitted to end users, electrical energy is increasingly being produced by small-scale generators located closer to points-of-use [3]. This distributed power generation center has made it easier to make use of all kinds of renewable energy sources and significantly reduced the energy transmission cost.

However, the inability to control the increased number of distributed energy sources can create huge difficulties in operating and controlling the distribution network. To solve this problem, the idea of microgrid is introduced [4]. A microgrid can be viewed as a "prosumer" (producer and consumer) [3]. It contains one or multiple kinds of renewable power generation centers and a single or a small group of energy users, and offers the possibility of coordinating the distributed resources in a more intelligent way so that they can behave as a controlled entity. In this way, distributed resources can provide their full advantages in a more consistent way.

Generally for each microgrid, as electricity must be used as it is being generated, the usual practice is to match supply to demand [5]. This is a challenging problem because of the following reasons: first, the power demand depends on exogenous factors and varies dramatically [6]; second, each energy user can have its own utility preference of energy usage at different time slots; third, due to the various types of power generation centers, the power generation cost will also vary as a function of time and weather factors, e.g., for a solar energy center, the power generation cost will be much cheaper in the day time than during the night time. Under a certain resource constraint, it is necessary to decide the amount of energy generation and consumption at each time so that the social welfare of the microgrid can be maximized [7].

In addition, the market idea has been in the heart of major roadmaps for microgrid structures [3-6]. The inter-connect network enables the surplus generation from one microgrid to be used to meet the local demand in its neighborhood. In the classic economic study, trade is always beneficial to both sides and thus should be encouraged [8]. By trading with each other, each microgrid can make use of its comparative advantage and achieve an increase of social welfare. Previous papers such as [3] have studied structure of the energy trading platform in smart grid neighborhoods. However, they failed to provide an analysis on some of the economic factors such as what is the optimal energy trading volume or what determines the relative energy price.

To give a detailed study on the above problems, two models are introduced in this paper. The first model deals with the local welfare maximization problem where a microgrid is considered as a closed economy group and decides the optimal power generation distribution in terms of time. In the second model, each microgrid can trade with its neighborhoods and thus achieve a welfare increase from making use of its comparative advantage on power generation during a certain period of time. A modified Ricardian model is used to study the economic factors in the trade process [9].

The remainder of this paper is organized as follows. In the next section, we present our local welfare maximization model for each microgrid. In section Section III, the electricity trade model is introduced for microgrid neighborhoods. Section IV reports the experimental results and the paper is concluded in Section V.

II. Model of Local Welfare Maximization

In the first model, we start with the assumption that a microgrid is a closed economy group in terms of energy (i.e., no energy trading is allowed) and decides the optimal energy generation distribution in terms of time. A slotted time model is assumed, i.e., all system cost parameters and constraints as well as energy generation and consumption decisions are provided for discrete time intervals of constant length. We use C_i and P_i to represent the energy generation and consumption levels (with a unified energy unit) at time slot i where $i \in \{1, 2, ..., T\}$. T is the total number of equal-sized time slots that we divide one *operating period* into. For example, if we set one operating period to be a day and $T = 24$, a day is divided into 24 time slots, each with duration of one hour.

Economists have agreed that users consume commonalities (such as energy) at each time because this energy consumption provides satisfaction, or *utility*, which represents the level of a kind of social welfare [9]. It is commonly modeled and verified in economic study that the relationship between the utility derived from the level of energy consumption at each time follows the form [9]:

$$U = \prod_{i=1}^{T} C_i^{\alpha_i}, \tag{1}$$

where α_i is the *preference factor* at each time slot and we have $0 < \alpha_i < 1$ for all i. A higher α_i means the users in this microgrid prefer to consume more energy at the corresponding time slot.

Equation (1) also reflects three characteristics of the relationship between the utility and energy consumption level:

1. As $\alpha_i > 0$, the utility is an increasing function of C_i, which means the overall satisfaction level will be increased if users can consume more energy at any time slot.

2. As $\alpha_i < 1$, the marginal utility, $\partial U/\partial C_i = \alpha_i C_i^{\alpha_i - 1} \prod_{j \neq i} C_j^{\alpha_j}$, is a decreasing function of C_i. This means if the energy consumption at one time slot is already at a very high level (i.e., much more than necessary), users' satisfaction level will not increase too much if we further increase the energy production level at that time. As a result, to satisfy the energy users in the microgrid, it is better to increase the energy production at the more "necessary" time slots, which is realistic.

3. The utility function drops to 0 if $C_i = 0$ for any $1 \leq i \leq T$, which means the level of satisfaction will be very low if power outage occurs during any period of time.

On the other hand, the generation centers should decide the energy generation distribution in terms of time so that the total utility of the microgrid is maximized. It is also commonly modeled in economics that under a given resource constraint, the energy generation *production possibilities* should follow the given inequation [9]:

$$\sum_{i=1}^{T} \beta_i P_i \leq I, \tag{2}$$

where β_i is the number of resource units that are needed to generate one unit of energy at each time slot and I is the total number of resource units that is allowed to use during one operating period. Notice that β_i is determined by the type of energy generation centers as well as the level of technology, and might not be constant for different i values.

Given the models for energy generation and consumption, we need to determine the relationship between the two. Unlike other common forms of energy such as chemical or kinetic, electrical energy should be used as it is being generated. If storage is required, electrical energy will typically be converted immediately into another form of energy such as potential, kinetic, or electrochemical[1][2]. In most of the recent microgrid structures, energy storage is not used because of the high cost[3][4]. In addition, as stated at the beginning of this section, the microgrid is assumed to be a closed economy group in terms of energy in this model. Therefore, no energy trade is considered in this model. Based on the above assumptions, the energy generation and consumption level should be the same at every time inside the microgrid, i.e., $C_i = P_i$ for each i.

Using the above definitions and assumptions, the local welfare maximization problem for one microgrid can be modeled as follows:

Local Welfare Maximization Problem for a Microgrid

Find the optimal energy generation P_i for $1 \leq i \leq T$.
***Maximize*:**

$$U = \prod_{i=1}^{T} C_i^{\alpha_i}$$

***Subject to*:**

$$\sum_{i=1}^{T} \beta_i P_i \leq I$$
$$P_i \geq 0 \quad \forall \quad 1 \leq i \leq T$$
$$C_i = P_i \quad \forall \quad 1 \leq i \leq T$$

The geometry optimization idea [12] can be used to numerically solve this problem.

Property 1: At the optimal solution point of the above local welfare maximization problem, we have:

$$\sum_{i=1}^{T} \beta_i P_i = I, \tag{3}$$

$$\frac{\partial U/\partial C_i}{\partial I/\partial P_i} = \frac{\alpha_i U}{\beta_i C_i} = D \quad \forall \quad 1 \leq i \leq T, \tag{4}$$

where D is the same value for all i, which results in the optimal solution given by:

$$P_i = C_i = \frac{\alpha_i}{\beta_i \sum_{m=1}^{T} \alpha_m} I \quad \forall \quad 1 \leq i \leq T. \tag{5}$$

***Proof*:** It can be easily determined that only when we make full use of the energy generation resources can we achieve the maximal utility function. In addition, assume at the optimal solution point, there exists a pair of time slots $\{i,j\}$ that $\frac{\partial U/\partial C_i}{\partial I/\partial P_i} > \frac{\partial U/\partial C_j}{\partial I/\partial P_j}$. We will be able to find a new possible solution with $C_i^{'} = C_i + \sigma/\beta_i$ and $C_j^{'} = C_j - \sigma/\beta_j$, where σ is a very small value with the same unit of total resource I. As we have $\frac{\partial U/\partial C_i}{\partial I/\partial P_i} > \frac{\partial U/\partial C_j}{\partial I/\partial P_j}$, it can be proven that $U(C_1, C_2, ..., C_i^{'}, C_j^{'}, ...) > U(C_1, C_2, ..., C_i, C_j, ...)$, which conflicts the optimality of the solution. As a result, the optimal solution will occur only when $\frac{\partial U/\partial C_i}{\partial I/\partial P_i}$ is the same for every i.

It can be observed from the solution that for a closed microgrid, we need to spend a lot of resources to generate a certain amount of energy at the low-efficiency power generation time, i.e., when β_i is large. In order to solve this problem and also make better use of its high-efficiency time, a microgrid can choose to trade with its neighborhood, which is discussed in the next section.

III. Model of Welfare Maximization with Neighborhood Trading

In international economic studies, countries engage in international trade because they are different from each other and both of them can benefit from the differences by reaching an arrangement in which each does the things it does relatively well [9]. Similarly, by building the inter-connect power network, each microgrid can perform energy trade with its neighborhood in order to achieve a welfare increase.

Basically, an energy trade contract contains two steps: First, both microgrids get together to make their energy generation decisions in order to achieve a maximal overall utility function; Second, both sides decide the distribution of the benefit from trade in a fair way. The second step can be performed using a fair benefit distribution law that the ratio between the utility function after trade and the maximal utility function before trade (i.e., $U_{trade}/U_{local,max}$) is the same for both sides, which is relatively easy and is not discussed in detail in this section because of space limit (but is used and shown in experimental result section). In this section, we focus on the first step to deal with the total welfare maximization problem for the two microgrids. A modified "Ricardian model" is used in this section [9]. In subsection A, we start with a simplified model with only two time slots in order to study the characteristics of the optimal solution of the welfare maximization problem. The model is then extended to a more general case in subsection B.

A. Trading with Two Time Slots

In this model, we assume that the target microgrid and its neighborhood has different energy generation cost as well as the resource. We denote the target microgrid's total resource by I_s, the energy generation at each time slot by $P_{s,i}$ and the number of resource units that are needed to generate one unit of energy at each time slot by $\beta_{s,i}$. Similarly, we have I_n, $P_{n,i}$ and $\beta_{n,i}$ for its neighborhood. As before, C_i denotes the total energy consumption at time slot i and α_i denotes the corresponding preference factor. The preference factor is assumed to be the same for both microgrids because they live close to each other.

Using the above definitions and assumptions, the total welfare maximization problem for these two microgrid can be modeled as follows:

Total Welfare Maximization Problem for Two Microgrids

Find the optimal energy generation $P_{s,i}$ and $P_{n,i}$ for $1 \le i \le T$.
***Maximize*:**

$$U = \prod_{i=1}^{T} C_i^{\alpha_i}$$

***Subject to*:**

$$\begin{aligned}
&\sum_{i=1}^{T} \beta_{s,i} P_{s,i} = I_s \\
&\sum_{i=1}^{T} \beta_{n,i} P_{n,i} = I_n \\
&P_{s,i} \ge 0, P_{n,i} \ge 0 \quad \forall \quad 1 \le i \le T \\
&C_i = P_{s,i} + P_{n,i} \quad \forall \quad 1 \le i \le T
\end{aligned}$$

Notice that we have simply used "=" instead of "≤" in the first two constraints because we have already proven that the maximal utility can be achieved only when each microgrid has made full use of its resources.

In this subsection, we focus on a simplified situation that $T = 2$. Economists have used the idea of *comparative advantage* to analyze the motive of trading. Comparative advantage refers to the ability of a party to produce a particular good or service at a lower marginal and opportunity cost over another. For energy generation centers in a microgrid, the comparative advantage comes from the ability of generating energy at a particular time slot at a lower opportunity cost over the energy generation centers in another microgrid. For convenience, assume we have re-labeled the time slots to make $\beta_{s,1}/\beta_{n,1} < \beta_{s,2}/\beta_{n,2}$. Based on the definition, the target microgrid has comparative advantage on energy generation at time slot 1, and the neighborhood microgrid has comparative advantage on energy generation at time slot 2. Notice that comparative advantage is determined by the ratio of $\beta_{s,i}/\beta_{n,i}$ instead of the absolute values, which means even though a microgrid has a higher energy generation cost at every time slot, it can still have comparative advantage at some of the time slots.

As we have stated before that energy trading enables both microgrids to make better use of its comparative advantage at a special time slot, we are able to analyze the optimal solution.

Property 2: At the optimal solution point of the above total welfare maximization problem with $T = 2$, if $\beta_{s,1}/\beta_{n,1} < \beta_{s,2}/\beta_{n,2}$, we have either $P_{n,1} = 0$ or $P_{s,2} = 0$ (or both).

***Proof*:** Assume we have both $P_{n,1} > 0$ and $P_{s,2} > 0$ at the optimal solution point, we will be able to find a new possible solution with $P'_{s,1} = P_{s,1} + \sigma_s/\beta_{s,1}$, $P'_{s,2} = P_{s,2} - \sigma_s/\beta_{s,2}$, $P'_{n,1} = P_{n,1} - \sigma_n/\beta_{n,1}$ and $P'_{n,2} = P_{n,2} + \sigma_n/\beta_{n,2}$, where σ_s and σ_n are a very small values with the same unit of total resource I_s or I_n. As $\beta_{s,1}/\beta_{n,1} < \beta_{s,2}/\beta_{n,2}$, there should exists a pair of $\{\sigma_s, \sigma_n\}$ that $\beta_{s,1}/\beta_{n,1} < \sigma_s/\sigma_n < \beta_{s,2}/\beta_{n,2}$. In this case, we will have both $P'_{s,1} + P'_{n,1} > P_{s,1} + P_{n,1}$ and $P'_{s,2} + P'_{n,2} > P_{s,2} + P_{n,2}$, which leads to a higher total utility and conflicts the optimality of the solution.

Based on this property, at least one microgrid will be specified in generating energy at one time slot at the optimal solution point. And thus the optimal solution of the total welfare maximization problem with $T = 2$ can be achieved by comparing the partial optimal solution at $P_{n,1} = 0$ or $P_{s,2} = 0$.

Notice that there is another special situation where $\beta_{s,1}/\beta_{n,1} = \beta_{s,2}/\beta_{n,2}$. It can be proven that there are multiple optimal solution points in this case and one of the optimal solution can be achieved when each of the two microgrids simply maximizes its own utility as in section II, which means none of the two microgrids can gain from trade. This case is not discussed in detail because of space limit but is verified in the experimental result section.

B. Trading with Multiple Time Slots

When we extend the problem to a more general case with $T > 2$, the idea of comparative advantage can still be used to analyze the characteristics of the optimal solution. Similarly for convenience, assume we have already re-labeled the time slots so that we have $\beta_{s,1}/\beta_{n,1} < \beta_{s,2}/\beta_{n,2} < \beta_{s,3}/\beta_{n,3} < ... < \beta_{s,T}/\beta_{n,T}$. We can also come up with the property as follows:

Property 3: At the optimal solution point of the total welfare maximization problem with $T > 2$, if $\beta_{s,1}/\beta_{n,1} < \beta_{s,2}/\beta_{n,2} < \beta_{s,3}/\beta_{n,3} < ... < \beta_{s,T}/\beta_{n,T}$, there exists a time slot t that we have $P_{n,i} = 0$ for $\forall 1 \leq i < t$ and $P_{s,i} = 0$ for $\forall t < i \leq T$. In other words, there is at most one (probably zero) time slot in which the energy is generated by both microgrids.

***Proof*:** When there already exists a time slot t with both $P_{n,t} > 0$ and $P_{s,t} > 0$, if there exists another time slot $1 \leq i < t$ with $P_{n,i} > 0$, we will have $\beta_{s,i}/\beta_{n,i} < \beta_{s,t}/\beta_{n,t}$ together with $P_{n,i} > 0$ and $P_{s,t} > 0$. We can find a better solution according to the proof of Property 2. Similarly, there should not exist another time slot $t < i \leq T$ with $P_{s,i} > 0$.

Assume we have already known the value of t, based on the analysis in section II, we also have the following property:

Property 4: At the optimal solution point of the above problem with $T > 2$, $\frac{\partial U/\partial P_{s,i}}{\beta_{s,i}}$ is the same for each $1 \leq i < t$ and $\frac{\partial U/\partial P_{n,i}}{\beta_{n,i}}$ is the same for $t < i \leq T$.

***Proof*:** Based on Property 3, we have $P_{n,i} = 0$ for $\forall 1 \leq i < t$ and $P_{s,i} = 0$ for $\forall t < i \leq T$, which means $C_i = P_{s,i}$ for $\forall 1 \leq i < t$ and $C_i = P_{n,i}$ for $\forall t < i \leq T$. Thus Property 4 can be concluded according to the proof of Property 1.

Property 3 and 4 can be used to determine the energy generation at all the other time slots when $P_{s,t}$ and $P_{n,t}$ are given:

$$P_{s,i} = \frac{\alpha_i}{\beta_{s,i}\sum_{m=1}^{t-1}\alpha_m}(I - \beta_{s,t}P_{s,t}) \quad \forall \quad 1 \leq i < t. \quad (6)$$

$$P_{n,i} = \frac{\alpha_i}{\beta_{n,i}\sum_{m=t+1}^{T}\alpha_m}(I - \beta_{n,t}P_{n,t}) \quad \forall \quad t < i \leq T. \quad (7)$$

As a result, given the value of t, the total welfare maximization problem with $T > 2$ can be simplified as follows:

Total Welfare Maximization Problem with a Given t

Find the optimal energy generation $P_{s,t}$ and $P_{n,t}$.
***Maximize*:**

$$U = \prod_{i=1}^{t-1}[\frac{\alpha_i}{\beta_{s,i}\sum_{m=1}^{t-1}\alpha_m}(I - \beta_{s,t}P_{s,t})]^{\alpha_i} \cdot$$

$$\prod_{i=t+1}^{T}[\frac{\alpha_i}{\beta_{n,i}\sum_{m=t+1}^{T}\alpha_m}(I - \beta_{n,t}P_{n,t})]^{\alpha_i} \cdot (P_{s,t} + P_{n,t})^{\alpha_t}$$

***Subject to*:**

$$0 \leq P_{s,t} \leq I_s/\beta_{s,t}$$
$$0 \leq P_{n,t} \leq I_n/\beta_{n,t}$$

There are only two variables in the above problem and they can be easily solved using geometry optimization with the help of matlab.

The remaining problem is to determine the value of t, and a straightforward algorithm is to switch t from 1 to T with a complexity of O(n). However, it can be proven that the maximal utility is a *concave function* of t. In addition, it can be guaranteed that the optimal t value is already achieved when we end up with both $P_{s,t} > 0$ and $P_{n,t} > 0$ in solving the above-mentioned optimization problem. Detailed proof is omitted in this paper because of space limit. Using these properties, we can terminate our algorithm once we have $U(t) \leq U(t-1)$ or both $P_{s,t}$ and $P_{n,t}$ are greater than zero. A special case is that at the optimal solution point, there is no time slot in which the energy is generated by both companies. This situation is already included in our algorithm and will end up with $P_{s,t} = 0$ or $P_{n,t} = 0$.

IV. Experimental Results

To demonstrate the effectiveness of the proposed solutions, cases corresponding to the aforesaid models are examined. The proposed solutions have been implemented using Matlab and tested for random cases.

In the first simulation, we focus on the case $T = 2$. The preference factor of each time slot is set to be 0.3 and 0.7. Before trading, both the target microgrid and the neighborhood microgrid are considered to be closed economic groups and maximize their own utility functions. When the microgrids open up to trade, they first get together to decide the optimal energy generation at each time slot so that the total welfare can be maximized. After that, they distribute the total energy consumption in a fair way, following the rule that the ratio between the utility function after trade and the maximal utility function before trade is the same for both sides. The model is tested for random cases with different energy generation cost and total resource combinations. The detailed simulation result is presented in Table I.

TABLE I
SIMULATION RESULTS FOR DIFFERENT CASES UNDER $T = 2$

case	grid	β_1	β_2	I	before trade		after trade		$\frac{U_{trade}}{U_{local}}$
					P_1	P_2	P_1	P_2	
1	s	1	2	10	3.0	3.5	3.0	3.5	1
	n	4	8	20	1.5	1.75	1.5	1.75	1
2	s	1	2	10	3.0	3.5	4.5	2.75	1.04
	n	6	8	20	1.0	1.75	0	2.5	1.04
3	s	1	2	10	3.0	3.5	10	0	1.08
	n	6	8	100	5.0	8.75	0	12.5	1.08
4	s	5	5	40	2.4	5.6	5.4	2.6	1.12
	n	8	4	40	1.5	7.0	0	10.0	1.12
5	s	1	5	50	15	7.0	30	4.0	1.33
	n	8	4	40	1.5	7.0	0	10.0	1.33
6	s	1	5	50	15	7.0	50	0	2.09
	n	8	1	40	1.5	28	0	40	2.09

It can be observed from the above table that in case 1, as we have $\beta_{s,1}/\beta_{n,1} = \beta_{s,2}/\beta_{n,2}$, there is no comparative advantage between the two microgrids. As a result, the optimal solution can be achieved when each of the two microgrids simply maximizes its own utility as and none of the two microgrids can gain from trade. From case 2 to case 6, as long as there is a difference between $\beta_{s,1}/\beta_{n,1}$ and $\beta_{s,2}/\beta_{n,2}$, both microgrids can make use of its comparative advantage and achieve a welfare increase from trading with each other. The higher this difference, the more they can gain from trade. In addition, compare case 2 and case 3, we can also observe that the total energy generation resource will also affect the cooperative energy generation decision for both microgrids. In case 2, the resource of the neighborhood microgrid is relatively limited so the target microgrid needs to generate energy at both time slots. But in case 3, both microgrids have enough resources so that each of them turns out to be specified in energy generation at its own advantageous time slot.

In the second simulation, we analyze the general case with total time slot $T = 6$. We set $\beta_{s,1}/\beta_{n,1} < \beta_{s,2}/\beta_{n,2} < \beta_{s,3}/\beta_{n,3} < ... < \beta_{s,6}/\beta_{n,6}$ and test our model under cases with total resource combinations. The final result of cooperative energy generation at each time slot is shown in Table II.

TABLE II
COOPERATIVE ENERGY GENERATION RESULTS UNDER $T = 6$

	grid	I	P_1	P_2	P_3	P_4	P_5	P_6	$\frac{U_{trade}}{U_{local}}$
1	s	8	8.0	0	0	0	0	0	1.34
	n	200	1.7	9.7	9.7	9.7	9.7	9.7	1.34
2	s	120	40	20	10	0	0	0	2.70
	n	120	0	0	0	10	10	10	2.70
3	s	200	39	20	9.8	4.9	3.3	0.1	1.37
	n	6	0	0	0	0	0	1.5	1.37

Our solution is verified in the above table. Case 1 and case 3 are the two extreme cases in which one microgrid is totally specified in energy generation at only one time slot, while the other microgrid generates energy at every time slot. In case 2, as the two microgrids have relatively the same amount of resources, energy turns out to be generated by one certain microgird with a comparative advantage at that time slot. No matter in which case, both of the microgrids can achieve a welfare increase after opening up to trade.

Another observation from the above experimental results is that even if a microgrid has a much better technology, i.e., $\beta_{s,i}$ is smaller than its neighborhood at any time slot, it will still benefit from trading with other microgrids. And also, a microgrid with less energy generation resources is more likely to be specified in generating energy at a single time slot.

V. CONCLUSION

Two models are presented in this paper to deal with the welfare maximization problems for microgrids. In each model, a microgird is considered to be a "prosumer" to have both energy generation and consumption. In the first model, a closed economy group is considered for each microgrid and optimal power generation distribution is solved. In the second model, the microgrid cooperates with its neighborhood through energy trading and a welfare increase is achieved for both sides. The idea of comparative advantage is used for microgrids in making decisions on energy generation. For each model, an efficient solution is presented. The accuracy and efficiency of our presented solutions are verified by experimental results.

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