# Equal Power Allocation Scheme for Cooperative Diversity

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Abstract-- In this paper, we present the Equal Power Allocation (EPA) algorithm for Power allocation and partner selection under a given constraint of outage probability. The proposed algorithm is used for a cooperative diversity system using Amplify-and-Forward scheme. We represent the problem with a new formulation to find the minimum total required power satisfying the outage probability constraint. We also present a low complexity algorithm for selecting the partner node among all candidate partners. We develop the analytical model and evaluate the results for some typical cases to demonstrate that the performance of the EPA algorithm. It is shown that the proposed algorithm achieves almost the same performance as the previously published algorithms while reducing its implementation complexity

*Index Terms*- Cooperative diversity, Amplify and Forward, Equal power allocation, Partner selection.

#### I. INTRODUCTION

Cooperative diversity is a technique that combats the slow fading and shadowing effect in wireless communication channel [1] and [2]. In this technique, the spatially distributed users cooperate with each other in transmitting the desired information to the destination. This creates a virtual array of antennas that can reduce the system power requirement and improve the transmission rate.

One of the most important problems in cooperative diversity is the strategy of power allocation among users. Most of the related works focus on the problem to allocate a constant power to the source and its partners to achieve the minimum value of outage probability. The power allocation for the decode and forward strategy, based on simulation and observation, has been studied in [3]. Power allocation based on constrained optimization method has been studied in [4] and [5]. We presented a practical power allocation algorithm based on the optimal power allocation strategy in [6].

Another important challenge in cooperative diversity is to decide how many partners and which one of the many possible candidates should be chosen to cooperate with the source [7], [8]. In [7], a partner selection algorithm in an opportunistic relaying form has been proposed. It is assumed that all of the candidates of cooperation are ready to cooperate and in each packet transmission, the best partner will cooperate.

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One of the recent approaches in cooperative diversity problem is minimization of power for constant rate which satisfies a constraint of outage probability or error probability [9]-[13]. In [9], the authors have expressed a short term power which is the minimum power that satisfies capacity constraint of the problem and use this for problem of constant mean of power. In [10], the authors focused on the problem of constrained minimization of power but a closed form solution was not presented. Lifetime maximization problem via cooperative nodes in wireless sensor networks is discussed in [11]. In that paper, the minimization of the total power of cooperating nodes has been studied to maximize the network life for a given error probability. In [12], the authors assume a case with two partners and solve the minimization of power in the entire network. The adaptive modulation technique is applied in [13] to improve the spectral efficiency of cooperative strategy and minimize the power consumption. In [14], we presented the optimal algorithm for minimizing the power in amplify and forward cooperative diversity (AFCD).

In this paper, we concentrate on the reduction of the power for transmission with one relaying scheme. We present this problem in case of equal power allocation between source and partners. We show that the reduction in transmit power by cooperation is very great and this approach can be used for life time maximization for sensor networks.

In this paper, we present Equal Power Allocation (EPA) algorithm in amplify and forward cooperative diversity in wireless channel. Both problems of partner selection and power allocation (finding the total required power for a source and given set of partners) with the constraint of outage probability are presented. We show that the results of the EPA algorithm are close to results of OPA algorithm. The simplicity of the EPA algorithm makes it more suitable for implementation.

In section II, we express the model of wireless channel and the cooperative strategy which is employed in this paper. We express the Equal Power Allocation (EPA) in section III. Partner selection in EPA scheme is presented in section IV. In section V the numerical evaluation and the results of the simulations are being expressed and we conclude this paper in section VI.

# II. SYSTEM MODEL

In this paper, we assume a slow, flat fading wireless channel. In the other word, the bandwidth of our signal is smaller than the coherence bandwidth of channel and the inverse rate of transmission is smaller than the coherence time of channel. Noting this assumption, the fading coefficient of channel can be assumed unchanged in a few transmission periods. We assume that the channel coefficient has Rayleigh distribution in small scale behavior. The large scale behavior of channel path loss is modeled with  $D^{-\alpha}$ , where D is the distance between transmitter and receiver and  $\alpha$  is a positive constant between 2 and 6.

Our cooperative diversity strategy is Amplify and Forward (AF) with orthogonal transmission. In this strategy, each node selects a few partners and the partners relay the received signals from the source to the destination. Each relay can be a source in other transmission time intervals. In this paper, we assume that the source can select each set of the candidate partners for cooperation, i.e. it has not any limitation in the node selection process.

The partner selection and power allocation strategies of the proposed algorithm are based on the information of the means of the channel coefficients, between source and partners and between partners and destination. Also the source is not aware of the full CSI of the channels. The receiver has the information of the instantaneous CSI of the channels and uses the maximum ratio combining (MRC) to detect the source information from the signals of source and partners.

#### **III. EQUAL POWER ALLOCATION SCHEME**

In this section, we want to present the behavior of the outage probability of the transmission of one source with the given set of partners. We assume that s,  $r_i$  and d denote the source, i<sup>th</sup> partner and destination nodes, respectively and the distance between a and b nodes is represented by  $d_{ab}$ . Also, we use the normalized distance (with respect to  $d_{sd}$ ) with the symbol  $D_{ab}$  and the symbol P represents the transmit power of the nodes.

According to [2] and [5], the achievable rate for transmission of the source with m orthogonal AF partners in bits per second per Hertz is obtained from (1).

$$I = \frac{1}{m+1} \log(B_0 + \sum_{i=1}^{m} \frac{A_i B_i}{A_i + B_i + 1})$$
(1)

Where  $B_0$  and  $B_i$  denote the SNR of the link between source and destination and the SNR of the link between i<sup>th</sup> partner and destination, respectively. Symbol  $A_i$  denotes the SNR of the link between source and i<sup>th</sup> partner and *m* denotes the number of partners. Each of  $A_i$ ,  $B_i$  and  $B_0$  random variables have an exponential distribution because we have assumed that the amplitude of the channel coefficient has a Rayleigh distribution.

The outage probability of (1) is not obtained easily in general case. The behavior of term of the logarithm in (1) and its PDF is presented in [15]. The obtained PDF of the logarithm term has a

complex form and based on this we can not find the proper allocation of power values between source and the set of partners. According to [4] and [5], we try to find the total allocated transmit power based on the approximation of the outage probability.

To explain the outage probability behavior of (1), we can approximate each m + 1 terms of information by exponential distribution with mean of (2) and approximate the outage probability by first term of Taylor series expansion which can be derived from the moment generating function technique. By this manner, the approximation of the outage probability has the form of (3).

$$\lambda_i = \frac{1}{N_0} \frac{\frac{P_s}{d_{sr_i}^{d_s}} * \frac{P_{r_i}}{d_{r_id}^{d_s}}}{\frac{P_s}{d_{sr_i}^{d_s}} + \frac{P_{r_i}}{d_{r_id}^{d_s}}}$$
(2)

 $P_{out} = Prob\{I < R\}$ 

$$\cong \frac{\left(\left(2^{(m+1)R}-1\right)N_{0}\right)^{m+1}}{(m+1)!} * \frac{d_{Sd}^{\alpha}}{P_{S}} * \prod_{i=1}^{m} \frac{\frac{P_{S}}{d_{Sr_{i}}^{\alpha}} + \frac{Pr_{i}}{d_{r_{i}d}^{\alpha}}}{\frac{P_{S}}{d_{Sr_{i}}^{\alpha}} \frac{Pr_{i}}{d_{r_{i}d}^{\alpha}}}$$
(3)

Where  $P_s$  and  $P_{r_i}$  denote the transmit powers of the source and i<sup>th</sup> partner. This approximation has high accuracy in high SNR, because we remove 1 from the deficit terms in approximation. This approximation is equal to outage approximations of [4] and [5] which are used for optimal power allocation with the constraint on the total transmit power.

We want to obtain the minimum total required transmit power for the source and a given set of partners with a constraint on the outage probability in EPA. In general case this problem is an optimization problem of (4).

$$\min P_s + \sum_{i=1}^m P_{r_i} \tag{4}$$

$$s.t.P_{out}\left(P_s + \sum_{i=1}^{m} P_{r_i}\right) \le P_{out-th}$$

$$\tag{4-1}$$

$$P_{max} \ge P_s , P_{r_i} \ge 0 \tag{4-2}$$

Where  $P_{out-th}$  denotes the maximum permitted outage probability. We solve this problem in [14] by KKT method. The proposed solution for (4) has an iterative form for both algorithms of power allocation and partner selection. In this paper we use EPA scheme and find the total required power to satisfy the outage probability constraint and to compare the results with complex OPA scheme.

If we rewrite the term of produced SNR of the  $i^{th}$  partner in (2), which we have assumed an equal power for both source and partner, then we can simplify it to (5).

$$\lambda_i = \frac{P_s d_{sd}^{-\alpha}}{N_0} \left( D_{r_i d}^{\alpha} + D_{sr_i}^{\alpha} \right) = \lambda_0 \left( D_{r_i d}^{\alpha} + D_{sr_i}^{\alpha} \right)$$
(5)

Where  $P_s$  denotes the source and partner transmit power.

Now, if we apply (5) into the approximation of the outage probability, the form of the outage probability changes to (6).

$$P_{out} \simeq \frac{\left(2^{(m+1)R}-1\right)^{m+1} \prod_{i=1}^{m} \left(D_{r_i d}^{\alpha} + D_{sr_i}^{\alpha}\right)}{(m+1)!} \left(\frac{N_0 d_{sd}^{\alpha}}{P_s}\right)^{m+1}$$
(6)

So, to satisfy the outage probability constraint in (4-1),  $P_s$  must satisfies the (7) inequality and the total required power of the source and given set of partners must satisfy the (8).

$$P_{s} \geq \left(2^{(m+1)R} - 1\right) N_{0} d_{sd}^{\alpha} \sqrt[m+1]{\frac{\prod_{i=1}^{m} \left(D_{r_{id}}^{\alpha} + D_{sr_{i}}^{\alpha}\right)}{P_{out-th}(m+1)!}}$$
(7)

$$P_T \ge (m+1) \left( 2^{(m+1)R} - 1 \right) N_0 d_{sd}^{\alpha} \sqrt{\frac{\prod_{l=1}^m \left( D_{r_ld}^a + D_{sr_l}^a \right)}{P_{out-th}(m+1)!}}$$
(8)

The right side of (8) is the minimum of the total required power for source and the set of partners which will satisfy the outage probability constraint in EPA scheme. By the numerical evaluation we demonstrate that the derived total power in most cases is near to optimal total required power which is obtained from iterative algorithm or more complex algorithms. It is shown that if the nodes in the network have low processing power, then the EPA is preferred to OPA scheme without great reduction in performance of cooperation.

We noted that the source in this scheme should know the term of  $D_{r_i d}^{\alpha}$  and  $D_{sr_i}^{\alpha}$  for each cooperative partner. These terms are easily obtained from the mean of the received term of power in partners and destination and must be sent to the source with feedback.

## IV. PARTNER SELECTION IN EPA

In this section we will present an algorithm which selects the best set of partners from all candidate partners in EPA scheme. We have presented the optimal partner selection algorithm for AFCD in OPA scheme in [14]. The EPA scheme has a different form of solution with respect to the minimization problem of power and based on this, selecting the best set of cooperative partners from all candidate partners has a different algorithm in comparison of OPA partner selection scheme. For example, we will show that the total required power in EPA scheme for one cooperative partner, in case of a cooperative node located close to the source, is equal to the total required power of a case with a cooperative node located close to the destination. But we should be careful the total required power of the second case is smaller than the first case in OPA scheme. It shows that the second cooperative partner is preferred to the first cooperative partner in OPA scheme but there is not any difference when we select each of these nodes in EPA scheme.

As discussed before, the most important parameter of one partner in EPA scheme is (9). If it has a large value for cooperative nodes, the required power in (7) and total required power in (8) are large and vice versa. So, we can rank the candidate cooperative nodes for cooperation by parameter r in (9) and select the best set of partners based on this.

$$r_i = D_{r_i d}^{\alpha} + D_{s r_i}^{\alpha} \tag{9}$$

The value of r for the cooperative partners located in different position is shown in figure 1. In this figure, the source and destination nodes are located in (x, y) = (0,0) and (x, y) = (1,0), respectively and we assume that  $\alpha = 2$ .

This figure shows that the nodes which are located near to line between the source and destination have the best r and best performance in EPA scheme. Also the best node for cooperation is the middle point of this line.



Fig. 1. Value of r for different positions

If the source node has not a limitation in selecting every partner set from all candidate partners, it can rank the candidate partners by (9) and chooses the nodes with the minimum rvalue. The remaining problem is calculating the number of cooperative partners in the set which we have presented in [6]. This number is a function of network parameters (for example constraint outage probability and R) and the state of the candidate partners. Based on this, we can not predict the number of partners in the best set to minimize the total required power in EPA scheme and must find it in specific problems. In bellow, we present an algorithm to find the best set of partners with minimum required power in EPA scheme.

In this algorithm, the partner nodes are added separately to the best set of partners and then the total required power is calculated. If it is decreased with respect to the previous total required power, the next rank of candidate nodes is added, else the last added partner is removed from the best set and it selects the candidate partners of the set for cooperation. The complete flowchart of this algorithm is shown in figure 2.

We note that the presented algorithm in partner selection of the OPA scheme (in [14]) has an iteration for every steps of finding the optimal set of partners and this shows that the algorithm of partner selection of the EPA scheme has lower complexity than the OPA algorithm.



Fig. 2. The flowchart of the algorithm of the partner selection in EPA scheme

#### V. SIMULATION RESULTS AND NUMERICAL EVALUATION

In this section we compare the power performance of the cooperation with EPA and OPA schemes and non-cooperate scheme. We probe the results of partner selection of the EPA scheme and compare it with the partner selection of the OPA scheme and at last we show the impact of the network parameters in the calculating the best number of cooperative nodes.

### A. Performance of EPA Scheme

We assume that the candidate partners of transmission of the source to destination are placed according to figure 3.



Fig. 3. Position of the candidate partners

In EPA scheme, the best nodes for cooperation is  $n_i$  and  $n_j$ . In figure 4 and 5, the total required power for EPA, OPA and non-cooperative schemes are plotted in case of candidate partner set of  $\{n_4\}$  and  $\{n_4, n_{10}\}$ . In these figures we assume that  $\alpha = 2$ ,  $d_{sd} = 100m$ ,  $N_0 = 1e - 4$  and R = 2 b/s/Hz.

These figures show that the order of diversity of both EPA and OPA schemes are equal, but the total required power for EPA scheme has a loss of 0.2 dB in figure 4 and about 1 dB in figure 5. This is because of non-optimality of the power allocation between the partners and source in this scheme. We note that this loss is related to the state of the partners set. The difference between the required power for cooperative and noncooperative schemes become larger by decreasing the constraint of outage probability, because of the diversity's order (2 in figure 4 and 3 in figure 5).



Fig. 4. Total required power for EPA, OPA and non-cooperative schemes with candidate partner set of  $\{n_4\}$ 



Fig. 5. Total required power for EPA, OPA and non-cooperative schemes with candidate partner set of  $\{n_4, n_{10}\}$ 



Fig. 6. Performance of the partner selection algorithm for EPA and OPA scheme

Figure 6 shows the comparison between the performances of the presented algorithm of partner selection in the EPA scheme and the algorithm of optimal partner selection for different outage probability constraint. The network parameters are like to previous figures. In this figure the source can select every partners of figure 3 based on the presented algorithm for partner selection.

In figure 6, changes of the slope of the curves are related to changes of the number of cooperative nodes. These changes and the best set of partners for EPA and OPA schemes are shown in table I.

This table shows that the selected set of partners of EPA algorithm differs from the selected set of the OPA scheme, but

figure 6 shows that the maximum loss of performance of the EPA scheme is 0.8 dB. So, algorithm has an acceptable performance with low complexity.

THE BEST SET	OF PARTNERS FOR EPA ANI	D OPA SCHEMES
Pout-th	OPA Set	EPA Set
1e-5 to 3e-5	$\{n_9, n_{10}, n_5\}$	$\{n_4, n_{10}, n_1\}$
4e-5 to 7e-5		$\{n_4, n_{10}\}$
7e-5 to 3.4e-3	$\{n_9, n_{10}\}$	
3.5e-3 to 6e-3		(m_)
7e-3 to 3.1e-2	{n <sub>9</sub> }	{ <i>n</i> <sub>4</sub> }

 TABLE I

 THE BEST SET OF PARTNERS FOR EPA AND OPA SCHEMES

#### B. The Best Number of Partners

Table I shows that the number of partners is related to many parameters like network parameters (outage probability constraint and *R*), algorithm of power allocation between source and partners and the state of the candidate partners. Based on this to relax the dependency of this number to state of the candidate partners, we assume that the source has infinite candidate partners in one mediocre location  $([D_{sr_i}, D_{r_id}] = [0.1, 0.9])$  and the source can select these partners without any limitation. For this case the number of cooperative partners of the best set is shown in figure 7 for different network parameter conditions with EPA scheme.



Fig. 7. Best number of partners in EPA scheme

This figure shows that the best value for the number of partners in EPA scheme is decreased when R or the outage probability constraints increases and vice versa.

#### VI. CONCLUSION

We presented the Equal Power Allocation (EPA) algorithm that finds the minimum required power for each set of partners under a given outage probability constraint. We also presented a novel algorithm for partner selection in EPA scheme. The algorithm is simple and yet can achieve almost the same results that are derived by a much more complex optimal partner selection algorithm. We showed that when EPA algorithm is used as the basis of partner selection, the best number of cooperative partners is decreased by increasing the transmission rate and outage probability constraint.

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