Concurrent Extraction of Partial Kernels with Timing Consideration

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Outline

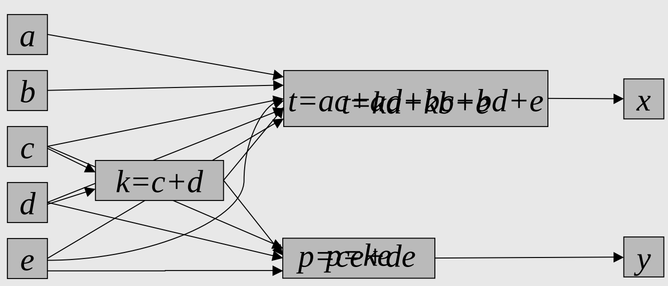
- Introduction
- Concurrent Extraction of Partial Kernels
- Timing Consideration
- Experimental Results
- Conclusion

Introduction

- Logic optimization objectives
  - Area
    - Estimate: Literal count
  - Delay
  - Power consumption
  - Testability
- Logic optimization techniques
  - Two-level
  - Multi-level
    - Logic Extraction

Logic Extraction

- Exploiting commonality among different expressions to reduce literal count



Literal count: ~~13~~ 9

Kernels (*Brayton & McMullen, '82*)

- Kernel
  - A cube-free divisor resulting in single-cube quotient
  - Much fewer than general divisors
- Theorem:
  - Two expressions *f* and *g* have a common multiple-cube divisor iff some kernel of *f* and some kernel of *g* have more than one cube in common.
- Drawback
  - Number of kernels can grow exponentially.

Fast Extract (*Rajski et al., '92*)

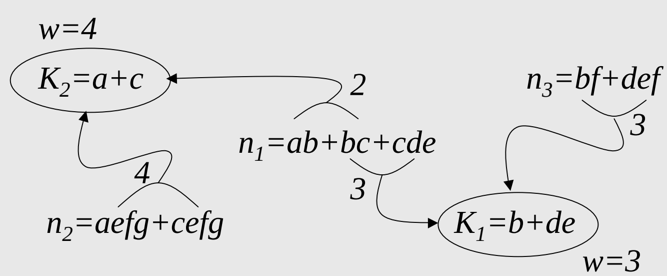
- Only consider double-cube (and two-literal single-cube) divisors.
- Objects of size 2 ⇨ Polynomial domain
- Theorem:
  - Two expressions have a common multiple-cube divisor iff they have a common double-cube divisor

Fast Extract – cont’d

for each node n_l in the network:
  for each cube pair (c_i , c_j) in n_l:
    k_k = kernel corresponding to (c_i , c_j);
    update the weight for k_k;
while there exists a kernel with positive weight:
  select the kernel k_k with maximum weight;
  extract k_k from all its cube pairs;
  update other kernels’ weights;

- Drawback: *Greedy* and *total*
- Improvement: *Concurrent partial* extraction

Motivational Example



Fast Extract:
  Extract K_2 from n_1, n_2

  Saving: *4* literals

Optimum Solution:
  Extract K_2 from n_2
  Extract K_1 from n_1, n_3
  Saving: *5* literals

Definitions

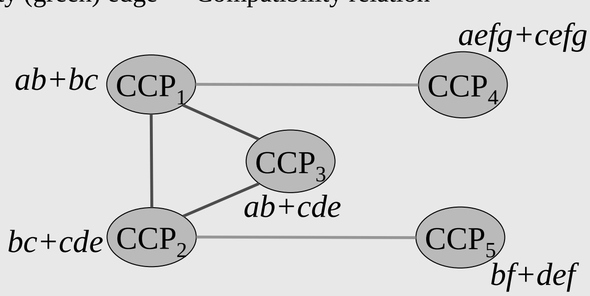
- Candidate Cube Pair (CCP):
  - Any two cubes belonging to the same node
  - Associated with a double-cube kernel
- Compatible CCPs:
  - Two CCPs corresponding to the same kernel
  - Example: (*ab*, *bc*) ~ (*aefg*, *cefg*)
- Conflicting CCPs:
  - Two CCPs having a common cube
  - Example: $n_1=ab+bc+cde$

$a+c$ $b+de$

Definitions – cont'd

- CCP Graph
  - Vertices ⇔ CCPs
  - Conflict (red) edge ⇔ Conflict relation
  - Compatibility (green) edge ⇔ Compatibility relation

$n_1=ab+bc+cde$
$n_2=aefg+cefg$
$n_3=bf+def$



Definitions – cont'd

- CCP Class
  - All CCPs corresponding to the same kernel
  - Makes a clique with compatibility edges
  - Sub-clique ⇔ Partial extraction
- Gain
  - Defined for each CCP, is the literal saving upon substitution of a variable for the kernel in that CCP
- Cost
  - Defined for each CCP class (clique), is the number of literals in kernel
- Value:
  - Defined for each clique/sub-clique, is:

$$value(c) = \sum_{v_i \in c} gain(v_i) - cost(c)$$

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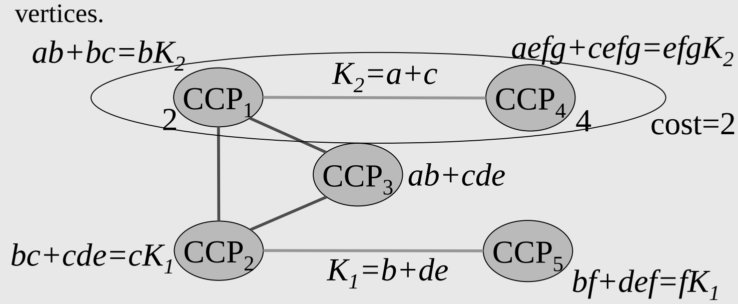
Problem Formulation

- Concurrent Partial Kernel Extraction (CPKE): Find a set of cliques (not necessarily maximal) in the CCP graph to maximize the total value in such a way that no conflict edge exists between two selected vertices.



- Theorem: CPKE problem is NP-Hard.

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Proposed Approach

- Formulate the CPKE problem as the maximum–weight independent set problem (MWIS).
  - MWIS is NP-Hard, however, it is a well-studied problem and a number of effective heuristics exist for it.
  - MWIS formulation also enables us to consider other cost functions.

Yet More Definitions!

- CCPs are of two types:
  - Essential (green): no conflict
  - Optional (red): in conflict with some CCP
- Partial CCP class
  - Subset of a CCP class containing all essential CCPs
  - Corresponds to a partial kernel extraction
  - Associated with a value as: $value = \sum gain - cost$
- Conflicting Partial CCP classes
  - Partial CCP classes that have some CCP in conflict

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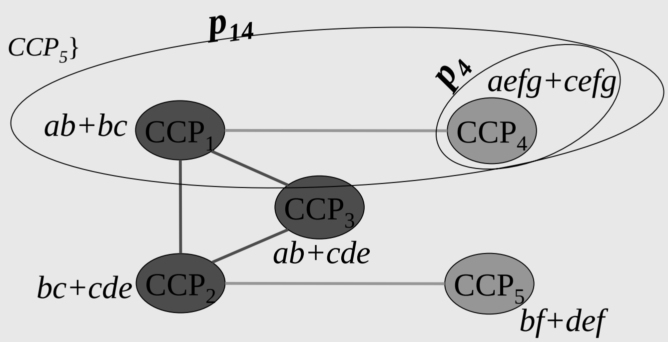
Example

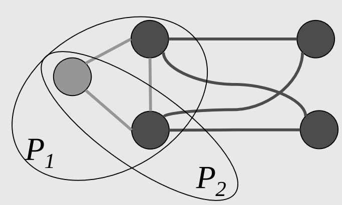
$p_3 = \{CCP_3\}$
$p_4 = \{CCP_4\}$
$p_{14} = \{CCP_1, CCP_4\}$
$p_5 = \{CCP_5\}$
$p_{25} = \{CCP_2, CCP_5\}$



Last Definition!

- Dominating partial CCP class

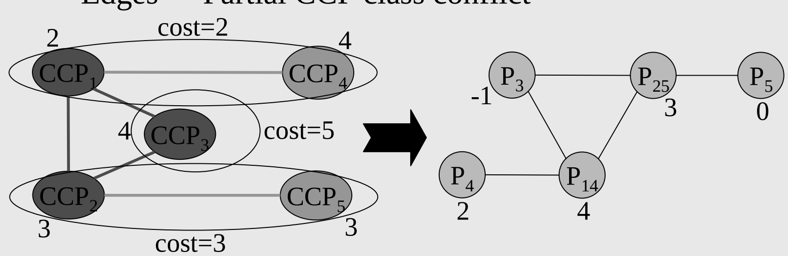
$P_2 \subset P_1$



- Prime partial CCP class
  - A non-dominated partial CCP class

New Problem Formulation

- Build Conflict Graph:
  - Vertices ⇔ Prime partial CCP classes
  - Edges ⇔ Partial CCP class conflict



- Find max-weight independent set of vertices

Timing Consideration

- Objective:
  - Minimize literal count while not increasing delay
  - Trade off delay to further decrease area
- Delay estimate:
  - Unit delay model
  - Unit-Fanout delay model
  - Library delay model

Timing Consideration

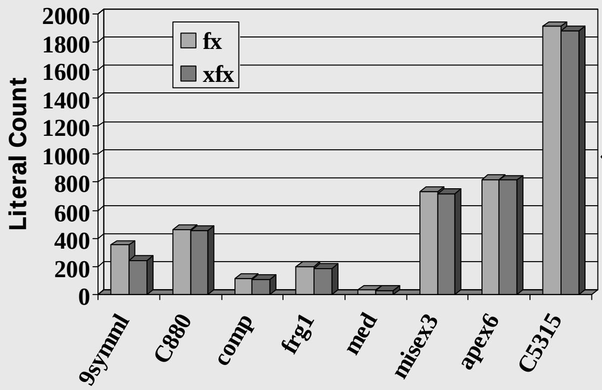
```
Algorithm Timing_Analyzer (S)
    for each prime partial CCP class p in S:
        for each CCP in p:
            mark the node corresponding to CCP;
    traverse network in DFS order:
        arrival (n) = 1 + max (arrival ( fanin (n) ) );
        if n is marked then
            arrival (n) = arrival (n) + 1;
    return max ( arrival ( PO ) );
```

- To speedup, operate on critical sub-network

Timing Consideration

- Extraction E_1 is superior to E_2 if E_1 results in:
  - Larger or equal literal saving and smaller level increase, or
  - Larger literal saving and smaller or equal level increase
- To prune a sub-tree of search space, use upper-bound for literal saving and lower-bound for level increase.

Experimental Results - I

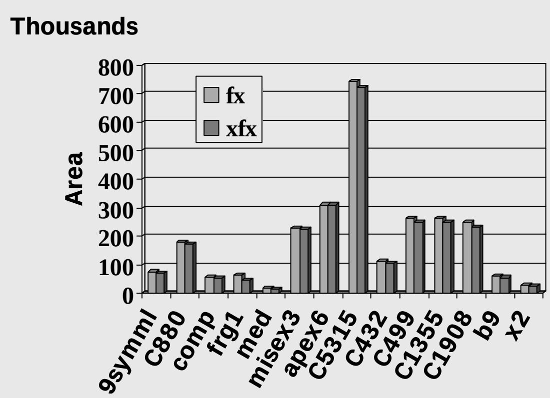


Literal count improvement

Max: 30%

Ave: 5%

Experimental Results - II

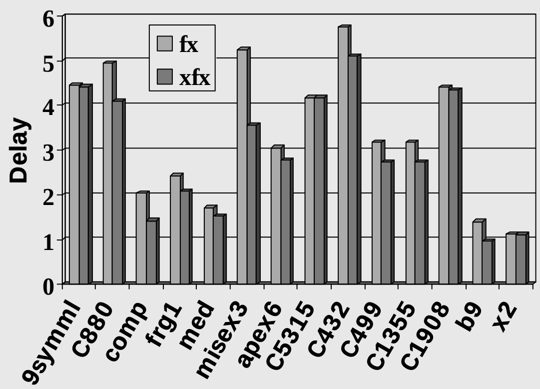


Area improvement

Max: 27%

Ave: 5%

Experimental Results - III



Delay improvement

Max: 32%
Ave: 13%

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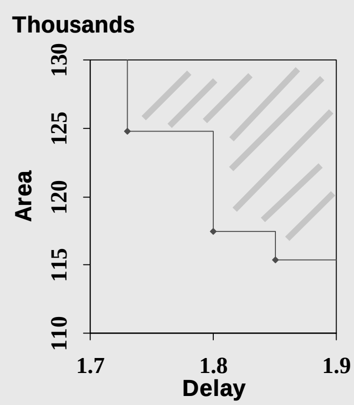
Experimental Results - IV

Benchmark: *9symml*

Area:	124743	117454	115337
Delay:	1.73	1.80	1.85



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Conclusion

- Introduced and formally defined the problem of concurrent extraction of partial kernels.
- Proved the problem to be NP-Hard.
- Formulated the problem as the MWIS problem in a graph.
- Proposed a method to control delay increase while doing extraction.