Negotiation-Based Task Scheduling and Storage Control Algorithm to Minimize User's Electric Bills under Dynamic Prices

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Abstract*—Dynamic energy pricing is a promising technique in the Smart Grid to alleviate the mismatch between electricity generation and consumption. Energy consumers are incentivized to shape their power demands, or more specifically, schedule their electricity-consuming applications (tasks) more prudently to minimize their electric bills. This has become a particularly interesting problem with the availability of residential photovoltaic (PV) power generation facilities and controllable energy storage systems. This paper addresses the problem of joint task scheduling and energy storage control for energy consumers with PV and energy storage facilities, in order to minimize the electricity bill. A general type of dynamic pricing scenario is assumed where the energy price is both time-of-use and power-dependent, and various energy loss components are considered including power dissipation in the power conversion circuitries as well as the rate capacity effect in the storage system. A negotiation-based iterative approach has been proposed for joint residential task scheduling and energy storage control that is inspired by the state-of-the-art Field-Programmable Gate Array (FPGA) routing algorithms. In each iteration, it rips-up and re-schedules all tasks under a fixed storage control scheme, and then derives a new charging/discharging scheme for the energy storage based on the latest task scheduling. The concept of congestion is introduced to dynamically adjust the schedule of each task based on the historical results as well as the current scheduling status, and a near-optimal storage control algorithm is effectively implemented by solving convex optimization problem(s) with polynomial time complexity. Experimental results demonstrate the proposed algorithm achieves up to 64.22% in the total energy cost reduction compared with the baseline methods.

I. INTRODUCTION

The advancement of the power system necessitates to address numerous issues, including carbon emissions, environmental sustainability, the increasing and imbalanced power demands, and the consumers' demands for better supply quality and higher reliability [1][2]. In order to nurture a low-carbon power system, renewable energy markets, industries, and policy frameworks have evolved rapidly in recent years [3]. The *photovoltaic* (PV) system, that converts solar radiation into electricity, is considered to be one of the most promising alternative power sources and is believed to play an important role in the process of transferring to the future low-carbon power system [3]. During the five-year period 2008–2012, the PV installation capacity has been growing at an average annual rate of 60%, and more importantly, interests in community-owned and self-generation PV systems (i.e. off-grid PV systems) have arisen rapidly as well [3]. The falling prices of off-grid PV systems have resulted in the emergence of decentralized PV power generation that is increasingly becoming a key power source especially among residential energy users [4]. This necessitates the consideration of the optimal control of off-grid PV power generation.

Unlike traditional power sources such as fossil fuels, PV power generation has intermittency issues, i.e., a PV system cannot generate electrical energy during night without sunshine or daytime in cloudy or rainy weather. Besides the intermittency issues, the peak PV power generation also varies according to various factors such as solar angle and weather conditions. The intermittency and variability of PV power generation together with the uncertainty of load power consumption lead to a mismatch between the peak PV generation time (usually around noon) and the peak load consumption time (usually during night) in a day [5]. Consequently, this timing mismatch prevents consumers from efficiently utilizing peak PV generation to compensate for peak power consumption, and as a result, the capability of power shaving using off-grid PV systems is significantly limited. In order to overcome the above challenge, energy storage is incorporated with off-grid PV systems so that consumers can store energy from the PV system when it has surplus energy generation and utilize the energy later during peak demand hours.

Apart from renewable energy sources and energy storages, further enhancement in grid efficiency can be achieved by embedding intelligence into the power grid [2]. The next-generation intelligent grid is known as the *smart grid* which has emerged as a well-planned plug-and-play integration of small power grids that will be interconnected through dedicated highways for command, data, and power exchange [2]. In a smart grid, the *Advanced Metering Infrastructure* (AMI) provides utilities with a two-way communication system to the meters that are deployed on the consumer side [2]. The various data gathered from these smart meters allow the utilities to apply embedded-control and decision-making protocols to improve the overall efficiency of the grid [6].

A major challenge in the smart grid is to match the power generation and the real-time electricity demand that varies dramatically due to exogenous factors such as time and season [1][6][7]. The utility companies are supposed to match the worst-case load profile, i.e., the peak power demand, in order to avoid potential power failures such as brown-outs and black-outs. Since 1982, the growth in peak load demand has exceeded transmission growth by almost 25% every year [1], indicating that the imbalance between electricity generation and consumption is becoming extremely severe. Without a smart grid technique, the U.S. government will have to invest hundreds of billions of dollars in new power plants over the next twenty years to meet the worst-case electricity load profile, leading to huge financial pressure, energy waste and potential environmental problems [6]. *Dynamic energy pricing* is a promising technique in the smart grid to alleviate the mismatch between electricity generation and consumption [1][6][7]. The key idea behind dynamic pricing is to assign different energy prices for consumers based on various factors such as time, season and consumers' real-time electricity consumption, which (i) incentivizes consumers to shape their power demands in order to lower the electric bills, and (ii) at the same time the reliability and efficiency of the overall grid are improved since most consumers have reshaped their demands driven by the dynamic energy pricing policy. An example in [8] shows that after the universal deployment of dynamic pricing policy in New York State, the societal welfare is improved by \$141-\$403 million per year.

Several techniques have been proposed in the context of minimizing user's electric bills under dynamic energy prices. The authors in [1] map this problem to the multiple knapsack problem which enables cheap and efficient solutions to the scheduling problem. To make the scheduling more realistic, the authors in [6] allow the scheduling of user's electricity-consuming applications (tasks) outside preferable time windows, but with an incurred inconvenience cost, and a power limit is imposed on each time unit so as to avoid outage.

A force-directed task scheduling approach is proposed in [6]. In [7], the authors formulate a real-time task scheduling problem with the objective function to minimize the overall electric bill in a smart building under dynamic energy prices. A heuristic algorithm using evolutionary computation method is proposed in [7]. All of the above-mentioned works [1][6][7] provide effective algorithms to deal with the task scheduling problem under dynamic prices, however, they have not taken the advantage of PV power generation and energy storage into consideration, which are increasingly important parts for residential smart grid users. On the other hand, reference work [5] develops an optimal control algorithm for energy storage systems in households equipped with PV modules and energy storage systems, but the detailed scheduling for each residential-level task is not considered in [5].

In this paper, we focus on the case of a residential smart grid user equipped with energy storage systems and PV modules. A realistic electricity price function is adopted for the smart grid user, which is comprised of a time-of-usage (TOU) price indicating the time-dependent unit electricity price and a power-dependent price that depends on the user's real-time power consumption. We propose a negotiation-based iterative cost minimization algorithm to minimize the user's energy cost by jointly scheduling the electricity-consuming applications (tasks) and controlling the charging/discharging of the energy storage system. The negotiation-based cost minimization algorithm is inspired by the negotiation-based FPGA routing methods [9][10]. In each iteration of the algorithm, it rips-up and re-schedules all tasks under a fixed storage control scheme, and then derives a new charging/discharging scheme for the energy storage based on the latest task scheduling results. The concept of congestion is introduced to dynamically adjust the schedule of each task based on the historical results as well as the current scheduling status, and a near-optimal storage control algorithm is effectively implemented by solving convex optimization problem(s) with polynomial time complexity. Experimental results demonstrate that the proposed algorithm can achieve significant energy cost reduction compared to baselines.

The remainder of the paper is organized as follows. Section II formally presents the system model, including energy storage model and dynamic price model. Section III shows the proposed negotiation-based cost minimization algorithm. Section IV reports the experimental results, and the paper is concluded in Section V.

II. System Model and Price Functions

In this paper, we consider a cooperative residential Smart Grid *user* who pays a unified electric bill for a group of tenants in a shared living space equipped with PV power generation systems and energy storage systems. Our objective is to minimize the electric bill of the user by controlling the scheduling of the user's electricity-consuming applications (tasks) and the charging and discharging decisions in energy storage systems.

A. Time Model and Task Model

A *slotted time* model is adopted in this paper, i.e., all system parameters, constraints and scheduling decisions are provided for discrete time intervals of constant and equal length. In this time model, the entire scheduling period is divided into T equal-sized time slots and each time slot has a duration denoted by D. Without loss of generality, we consider the task scheduling problem in one day that is further divided into 24 time slots, i.e., $T = 24$ and $D = 60$ minutes. The proposed algorithm can be easily extended to a different time scale such as a month, a season, etc.

For the user of interest, a number of electricity-consuming tasks are performed on a daily base, which reflect the electric loads of the residential user. Each task has an index i as its identification and the set of all task indices is denoted as $\mathcal{T} = \{1, \dots, N\}$, where N is the total number of tasks. Besides, each task $i \in \mathcal{T}$ has an earliest start time S_i, a deadline E_i and a duration D_i to complete the task, where S_i, E_i and D_i are specified by the user based on the electricity requirements at the beginning of the day. We assume the tasks are non-interruptible, i.e., each task executes from the scheduled start time that is denoted by λ_i until completion at $\lambda_i + D_i$ without discontinuity. In the previous work [7], tasks can only be scheduled inside the time window $[S_i, E_i]$. To make the scheduling problem more realistic, we allow each task i to be scheduled outside its preferable time window but with an incurred *inconvenience cost* I_i, which indicates the degree of inconvenience to the user. Of note, the tasks that must be executed within the time window $[S_i, E_i]$ are modeled by setting the inconvenience costs to infinite.

We denote the power consumption function of task i in time slot t by $p_i(t)$. Once task i starts to execute ($t = \lambda_i$), the power consumption of task i will follow a given profile (from time slot λ_i to $\lambda_i + D_i$) independent of the scheduled start time λ_i. Moreover, the power consumption of a task outside the scheduled execution time slots is zero. The earliest start time, deadline, duration, inconvenience cost and power profile for each task are assumed to be provided by the user at the beginning of the day. Figure 1 illustrates an example of task scheduling solution. The scheduling result of each task is represented by a bar, e.g., TV is scheduled to operate from 3 pm to 11 pm.

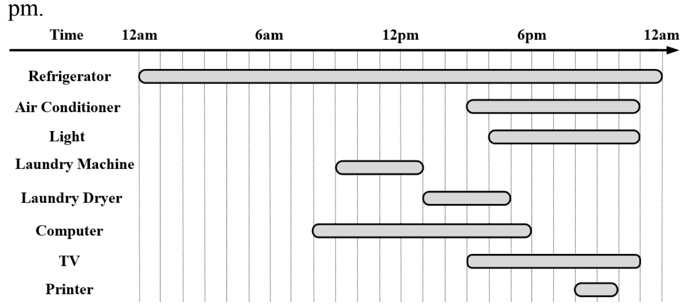

Fig. 1. A simple example of a household task scheduling solution.

B. System Diagram and Operation Modes

The block diagram of the residential smart grid user of interest is shown in the Figure 2. The PV module and the energy storage system are connected to the residential DC bus through unidirectional and bidirectional DC-DC converters, respectively. The DC bus is further connected to the residential AC bus via bidirectional AC/DC interfaces (e.g., inverters, rectifiers, and transformer circuitries). The residential AC loads on the AC bus correspond to residential tasks (as discussed in Section II.A) such as laundry machine, lighting and heating equipment. The AC bus is further connected to the state-level or national smart grid. We consider realistic power conversion circuits (i.e., the power conversion efficiency is less than 100%) in this work, and use η_1, η_2 and η_3 to denote the power efficiency of the converters between the PV module and the DC bus, the converters between the storage and the DC bus, and the AC/DC interface connecting the DC bus and the AC bus, respectively. Typical conversion efficiency values in the range of 85% to 95% are used in this paper.

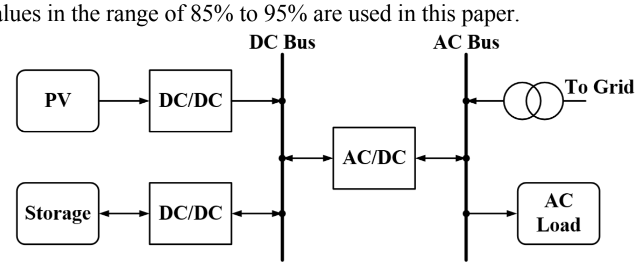

Fig. 2. Block diagram of residential user including PV module, storage system, residential load and the smart grid. The directions of arrows represent the directions of the power flow.

In the above-mentioned system, a part of energy is generated by the residential PV module, and we denote the PV power generation available to the user in time slot t by $P_{pv}(t)$ (i.e., the output power level of the PV module at time slot t). The residential load power consumption is denoted by $P_{load}(t)$, satisfying $\Sigma_{i=1}^{N} p_i(t)$. The output power level of the storage system in time slot t is denoted by $P_{st}(t)$, where a positive value of $P_{st}(t)$ means discharging from the storage, a negative value indicates that the storage is being charged and zero represents no charging or discharging operation. The instantaneous power consumption drawn from the grid in time slot t is denoted by $\omega(t)$, i.e., the *grid power*.

There are three *operation modes* in the system. In the first mode, the storage system is discharging, and thus, the residential loads are supplied simultaneously by the grid, the PV module and the storage. The condition to operate in the first mode is $P_{st}(t) \geq 0$, and the grid power in time slot t is calculated by

$$\omega(t) = \sum_{i=1}^{N} p_i(t) - \eta_3 \left(\eta_1 \cdot P_{pv}(t) + \eta_2 \cdot P_{st}(t)\right) \tag{1}$$

In the second mode, PV power generation is sufficient to supply the residential loads, and the surplus PV power is used to charge the storage. In this mode, there is power generated by the PV module flowing from the DC bus to the AC bus, and the condition to be in the second mode is given by $P_{st}(t) < 0$ and $\eta_1 \cdot P_{pv}(t) + \frac{1}{\eta_2} \cdot P_{st}(t) \geq 0$. The grid power in time slot t is calculated by

$$\omega(t) = \sum_{i=1}^{N} p_i(t) - \eta_3 \left(\eta_1 \cdot P_{pv}(t) + \frac{1}{\eta_2} \cdot P_{st}(t)\right) \tag{2}$$

In the third mode, the storage is being charged, and the PV module is insufficient for charging the storage. Thus, the storage is simultaneously charged by the PV module as well as the grid. In general, the residential system operates in this mode when PV power generation is insufficient and the energy price is relatively low, and hence it is reasonable to store some electrical energy for peak power consumption hours. The condition to operate in this mode is given by $P_{st}(t) < 0$ and $\eta_1 \cdot P_{pv}(t) + \frac{1}{\eta_2} \cdot P_{st}(t) < 0$. The corresponding grid power in time slot t is calculated by

$$\omega(t) = \sum_{i=1}^{N} p_i(t) - \frac{1}{\eta_3} \left(\eta_1 \cdot P_{pv}(t) + \frac{1}{\eta_2} \cdot P_{st}(t)\right) \tag{3}$$

For realistic concerns, we assume that the residential user cannot get paid by selling electric power back to the grid. To reflect this assumption, we set $\omega(t) = 0$ when $\omega(t) < 0$ in the problem formulation. This approach is equivalent to setting the price function to zero when $\omega(t) < 0$, and will not affect the optimization method of the cost minimization problem.

C. Energy Storage Model

In this paper, we consider a typical residential storage system made of lead-acid batteries or Li-ion batteries. In order to precisely analyze the system performance, an accurate model for the storage system is required. The most significant portion of power loss in lead-acid or Li-ion batteries is due to the *rate capacity effect* [11], i.e., capacity of a battery decreases with increase in discharging current [12].

When the rate capacity effect is accounted for, the storage model exhibits a non-linear relationship between the charging/discharging current (normalized to the *reference current*) and the degradation/increase rate of storage energy (normalized to the nominal capacity). We denote the increase/degradation rate of storage energy in time slot t by $P_{st,in}(t)$, and the terminal voltage of the storage system, which is denoted by V_{st}, is supposed to be (near-) constant. We apply the storage model developed in [5] to calculate the output power level of the storage by

$$P_{st}(t) = \begin{cases} V_{st} \cdot I_{st,ref} \left(\dfrac{P_{st,in}(t)}{V_{st} \cdot I_{st,ref}}\right)^{\beta_1}, if\ \dfrac{P_{st,in}(t)}{V_{st} \cdot I_{st,ref}} > 1 \\ P_{st,in}(t), if\ -1 \leq \dfrac{P_{st,in}(t)}{V_{st} \cdot I_{st,ref}} \leq 1 \\ -V_{st} \cdot I_{st,ref} \left(\dfrac{|P_{st,in}(t)|}{V_{st} \cdot I_{st,ref}}\right)^{\beta_2}, if\ \dfrac{P_{st,in}(t)}{V_{st} \cdot I_{st,ref}} < -1 \end{cases} \tag{4}$$

where $I_{st,ref}$ is the reference current of the storage system, and it is proportional to the storage's nominal capacity; β_1 and β_2 are coefficients ranging from 0.8-0.9 and 1.1-1.3, respectively.

The relationship between $P_{st}(t)$ and $P_{st,in}(t)$ given in (4) is further denoted by the function $P_{st}(t) = f_{st}(P_{st,in}(t))$, which is a concave and monotonically increasing function over the input domain $-\infty < P_{st,in}(t) < +\infty$. Because of the monotonicity property, the inverse function $P_{st,in}(t) = f_{st}^{-1}(P_{st}(t))$ is also a concave and monotonically increasing function over the input domain $-\infty < P_{st}(t) < +\infty$.

D. Price Model and Problem Definition

We adopt a realistic dynamic price model $\xi(t,\ \omega(t))$ comprised of a *time-of-use* (TOU) price that is determined by the time slot t, as well as a power-dependent price that depends on the grid power. The TOU price component is higher in the peak hour time slots than the off-peak periods, incentivizing the user to shift loads to off-peak hours. However, overreaching the goal is just as bad as not reaching it. If most users shift their tasks towards the off-peak hours, the power plants may fail to match the loads in those off-peak time slots. Thus, the power-dependent price component is set to be monotonically increasing with respect to $\omega(t)$, which prevents users from overly shaving loads towards the off-peak hours. By applying the above dynamic price model, the chance to see a power outage is much lower.

Using the above-mentioned definitions and system models, namely, task model, slotted time model, system diagram, energy storage model and dynamic price model, the overall cost minimization problem of the energy user can be modeled as follows:

Cost Minimization Problem for a Residential Energy User.

Find the optimal start time λ_i for $1 \leq i \leq N$ and the optimal $P_{st}(t)$ for $1 \leq t \leq T$.

Minimize:

$$Total\ Cost = \sum_{t=1}^{T} \xi(t, \omega(t)) \cdot \omega(t) + \sum_{i=1}^{N} I_i(\lambda_i) \tag{5}$$

where the inconvenience cost for task i is given as

$$I_i(\lambda_i) = \begin{cases} 0, & S_i \leq \lambda_i \leq E_i - D_i \\ I_i, & otherwise \end{cases} \tag{6}$$

Subject to:

$$-P_{st,MAX,c} \leq P_{st}(t) \leq P_{st,MAX,d} \tag{7}$$

$$0 \leq E_{st,ini} - \Sigma_{t'=1}^{t} P_{st,in}(t') \cdot D \leq E_{st,MAX}, \forall t \in [1, T] \tag{8}$$

where $P_{st,MAX,c}$ and $P_{st,MAX,d}$ are the maximum allowable amount of power flowing into and out of the storage system during charging and discharging, respectively; $E_{st,ini}$ is the initial energy storage at the beginning of the day and $E_{st,MAX}$ is the maximum energy storage capacity. Therefore, constraint (7) ensures that the charging and discharging current in the storage is in the allowable range, and constraint (8) ensures the stored energy is within the capacity of the storage throughout the entire scheduling period.

III. Negotiation-Based Energy Cost Minimization

A unified framework of task scheduling and energy storage control is developed for effectively solving the overall cost minimization problem. This section is organized as follows: part A explains the motivation to introduce the negotiation-based routing method; part B presents the proposed unified framework to find a suitable solution for the cost minimization problem.

A. *Motivation to Introduce Negotiation-Based Routing Method to the Cost Minimization Problem*

The objective of an FPGA routing algorithm is to find a feasible routing solution which connects all signal nets in the netlist using limited routing resources such as physical wires and switches so that the delay of critical paths is minimized. A routing result that requires even one more wire than the number of wires supported by a resource node is infeasible.

Reference works [9][10] propose the *Negotiation-Based Routing Algorithm* (NBRA) which provides the best solution known so far for the FPGA routing problem. In NBRA, a *congestion cost function* comprising a delay component as well as a *congestion value* that indicates the degree of resource sharing is introduced to each node, with the intention to balance the competing goals between minimizing the delay of critical paths and minimizing congestion in resource usage. Besides, NBRA is implemented in an iterative manner, and in each iteration, the congestion cost function gradually increases as the penalty for sharing resources. With the dynamically adapted congestion cost function, routing of each signal will avoid congested resources and meanwhile seek the shortest path. A feasible routing is achieved when there is no congestion in the circuit.

There are certain similarities between the FPGA routing problem and the cost minimization problem:

- Both problems are NP-hard [6][7][10]. Considering the computational complexity, it is reasonable to find a suitable solution effectively instead of the optimal one in both problems.
- It is impractical to find a suitable solution in one pass when the number of optimization variables is large. Instead, an iterative approach should be adopted in both problems, in order to gradually approach to a suitable solution.
- Sharing of routing resources causes congestion in the FPGA routing problem. In the residential cost minimization problem, tasks sharing a time slot lead to high power consumption and high energy price in that time slot. Thus, we want to reduce resource sharing in both problems.

Motivated by the above-mentioned similarities, the concept of congestion is introduced to the proposed algorithm to effectively solve the overall cost minimization problem.

B. *Negotiation-Based Cost Minimization Algorithm*

In this subsection, a *Negotiation-Based Cost Minimization* (NBCM) algorithm is proposed as the unified framework of task scheduling and energy storage control to find a good enough solution for the overall cost minimization problem.

We assume the parameters of the storage system such as $E_{st,MAX}$, $P_{st,MAX,d}$, $P_{st,MAX,c}$, $I_{st,ref}$ and V_{st} are given and remain unchanged during the entire day. At the beginning of the day, S_i, E_i, D_i, I_i and the power profile of each task i as well as the initial energy storage $E_{st,ini}$ are provided by the user, and the price functions are provided by the utility company. Besides, the PV power generation at each time slot is accurately predicted from the weather forecast and prediction algorithms presented in [13] at the beginning of the day. In the context of our problem formulation, a solution is a set of scheduled start times of all tasks in $\mathcal{T}$ that is denoted by $\Lambda_{task} = \{\lambda_1, \dots, \lambda_N\}$ together with a set of charging/discharging decisions of the storage system denoted by $\Lambda_{st} = \{P_{st}(1), \dots, P_{st}(T)\}$.

As mentioned in Section III.A, the proposed algorithm is implemented in an iterative manner, and finds a good enough solution when the total energy cost has not decreased for L iterations or the iteration number exceeds the maximum iteration number that is set to K. Each j-th iteration is comprised of two parts. In the 1st part, all previous task scheduling results are ripped up and each task is rescheduled based on the latest storage control scheme. Then in the 2nd part of each iteration, the storage control scheme is ripped up and new storage charging/discharging decisions will be made based on the latest task scheduling results generated in the 1st part of this iteration. Price functions are updated when a task is scheduled or an energy storage control decision is made within an iteration.

Figure 3 shows the pseudo code for the NBCM algorithm. In each iteration of the NBCM algorithm, it comprises of a *Negotiation-Based Task Scheduling* (NBTS) algorithm, which seeks the optimal start time for each task under fixed energy storage control decisions, and a *Negotiation-Based Storage Control* (NBSC) algorithm, which finds the optimal charging/discharging scheme of the storage system under fixed task scheduling results. Next we will introduce the NBTS algorithm and NBSC algorithm in details.

Algorithm *NBCM* ( )

Initialize tasks, PV, energy storage parameters, price functions, inconvenience cost, and constraints
Set iteration counter $j = 0$
Do
  $j = j + 1$ ⎫
  Rip up all tasks ⎬ → 1st part
  NBTS (j) ⎭
  Rip up all energy storage decisions ⎫
  NBSC (Λ_{task}) ⎬ → 2nd part
  Calculate overall energy cost by (5) ⎭
Until total energy cost is not decreased for L iterations or $j \geq K$
Return Λ_{task} and Λ_{st}

Fig. 3. Pseudo code for NBCM algorithm.

The NBTS Algorithm for Task Scheduling:

At the beginning of the j-th iteration of NBCM, all tasks are ripped up and the NBCM calls NBTS algorithm to reschedule all tasks one by one under the latest storage control scheme. To introduce the details in the NBTS algorithm, let $\omega_i^j(t)$ denote the total grid power consumed in time slot t after the i-th task has been scheduled where $t \in [1, T]$ and $i \in [1, N]$. When the NBTS algorithm schedules the i-th task, the NBTS scheduler has already known the scheduling decisions of tasks 1 to $i - 1$, the current price functions $\xi\left(t, \omega_{i-1}^j(t)\right)$ and grid power consumptions $\omega_{i-1}^j(t)$. We define the *Cost Increase* (CI) for the residential user after scheduling the i-th task as follows:

$$CI = \Sigma_{t=1}^{T}\left(\xi\left(t, \omega_i^j(t)\right) \cdot \omega_i^j(t) - \xi\left(t, \omega_{i-1}^j(t)\right) \cdot \omega_{i-1}^j(t)\right) + I_i(\lambda_i) \tag{9}$$

We have one important observation when scheduling task i in each iteration j: The time slots occupied by the previous $i - 1$ tasks are more likely to exhibit a higher energy price because the price function in each time slot t is monotonically increasing with respect to $\omega(t)$. We define a time slot to be *congested* when it is occupied by at least one task within an iteration. Hence, it is reasonable to avoid these congested time slots in the following steps in this iteration to minimize the overall energy cost. To quantify the degree of congestion in a time slot, we define an *intra-iteration congestion term* $R(t)$ as the number of tasks that have been scheduled to occupy time slot t within one iteration.

Besides avoiding congested time slots, it is also reasonable to guide the NBTS scheduler to fully utilize all the available PV power generations. Thus, an *inter-iteration congestion term* $H(t)$ is introduced, which is defined as the total times in the previous $j - 1$ iterations when PV power generation is not fully utilized in time slot t. Moreover, to resolve the problem that a number of suitable solutions are blocked when a task always occupies the same time slots in different iterations, we introduce another *inter-iteration congestion term* $h(i,t)$ that indicates the total times in the previous iterations when task i occupies time slot t (Initially, $h(i,t) = 0$ for all tasks in all time slots).

Of note, the inter-iteration congestion term is updated at the end of each iteration, whereas the intra-iteration term is updated within one iteration. After introducing the three congestion terms, we define a modified cost increase CI' in Eqn. (10). Please note that this modified cost increase CI' is the objective function for the NBTS scheduler to minimize when scheduling task i in iteration j.

$$CI' = \Sigma_{t=1}^{T}\left(\xi\left(t,\omega_i^j(t)\right)\cdot\omega_i^j(t) - \xi\left(t,\omega_{i-1}^j(t)\right)\cdot\omega_{i-1}^j(t)\right) (a\cdot h(i,t)+1)(b\cdot R(t) - c\cdot H(t)+1) + I_i(\lambda_i) \tag{10}$$

where a, b and c are positive weights of $h(i,t)$, $R(t)$ and $H(t)$, respectively[1]. After introducing the above congestion terms, the NBTS scheduler will avoid congested time slots within one iteration due to the term $b\cdot R(t)$, and try to use all available PV power generation because the term $c\cdot H(t)$ slightly lowers the cost in time slots where PV power generation is not used efficiently in previous iterations. Meanwhile, the term $a\cdot h(i,t)$ permanently increases when task i occupies time slot t in one iteration, and after several iterations, the scheduler may give up those time slots and try to search for other slots to schedule task i. After the N-th task is scheduled, the NBTS returns the schedule Λ_{task} to the main NBCM. This is the end of the 1st part of NBCM. The computational complexity of the NBTS algorithm is $O(NT^2)$. Figure 4 provides the pseudo code for the NBTS algorithm.

Algorithm *NBTS* (j)
```
Loop over all tasks i
¦ Loop over all time slots t ∈ [1, T − D_i + 1 ]
¦ ¦ Set the start time λ_i = t
¦ Choose the λ_i that minimizes objective function (10)
¦ Update the intra-iteration congestion term R(t)
Loop over all time slots t
¦ Update the inter-iteration congestion term H(t)
¦ Loop over all tasks i
¦ ¦ Update the inter-iteration congestion term h(i, t)
Return Λ_task
```

Fig. 4. Pseudo code for NBTS algorithm.

The NBSC Algorithm for Storage Control:

Then the NBCM continues to execute the 2nd part of the j-th iteration with the fixed task schedules. All energy storage control decisions are ripped up, and then NBCM calls the NBSC algorithm to derive a new charging/discharging scheme for the energy storage based on the latest scheduling results Λ_{task}. The optimization problem in NBSC, namely, the optimal storage control problem under fixed task scheduling is modeled as follows:

Storage Control Problem for a Residential Energy User.

Given the latest task scheduling results Λ_{task}.

Find the optimal $P_{st}(t)$ for $1 \le t \le T$.

Minimize:

$$Total\_Cost = \sum_{t=1}^{T}\xi\left(t,\omega(t)\right)\cdot\omega(t) + \sum_{i=1}^{N} I_i(\lambda_i) \tag{11}$$

where the grid power $\omega(t)$ is given by Eqns. (1), (2) or (3) based on the value of $P_{st}(t)$, $P_{load}(t) = \Sigma_{i=1}^{N} p_i(t)$ and $P_{pv}(t)$.

Subject to:

$$-P_{st,MAX,c} \le P_{st}(t) \le P_{st,MAX,d} \tag{12}$$

$$0 \le E_{st,ini} - \Sigma_{t'=1}^{t} P_{st,in}(t')D \le E_{st,MAX}, \forall t \in [1,T] \tag{13}$$

where constraints (12) and (13) are inherited from constraints (7) and (8), respectively. Since the scheduling results have been derived by the NBTS algorithm in this iteration and fixed during NBSC algorithm execution, the residential load power consumption in each time slot t, i.e., $P_{load}(t)$, can be calculated by $P_{load}(t) = \Sigma_{i=1}^{N} p_i(t)$, and the total inconvenience cost $\sum_{i=1}^{N} I_i(\lambda_i)$ can also be calculated. Therefore, in the objective function (11) in the NBSC problem formulation, (i) the residential load power consumption $P_{load}(t)$ is given and fixed, and (ii) the total inconvenience cost $\sum_{i=1}^{N} I_i(\lambda_i)$ is also fixed.

[1] The proposed algorithm can be applied to a user without off-grid PV energy supply by setting $c = 0$.

We have the following important observation in order to effectively solve the storage control algorithm via convex optimization techniques [14]:

Observation: The storage control problem will become a convex optimization problem with convex objective function and inequality constraints if (i) $P_{st,in}(t)$ for $t \in [1,T]$ are utilized as optimization variables (instead of $P_{st}(t)$), and (ii) assuming that $\xi\left(t,\omega(t)\right)$ is fixed for each time slot t.

Proof: Suppose that $P_{st,in}(t)$ for $t \in [1,T]$ are optimization variables and $\xi\left(t,\omega(t)\right)$ is fixed and independent of $\omega(t)$. We know that $P_{st}(t) = f_{st}\left(P_{st,in}(t)\right)$ is a concave and monotonically decreasing function over the input domain, and $\omega(t)$ is a convex and monotonically decreasing function of $P_{st}(t)$ (this is also valid when we set $\omega(t) = 0$ when $\omega(t) < 0$.) Hence, $\omega(t)$ is a convex function over the optimization variables $P_{st,in}(t)$ for $t \in [1,T]$ according to the rules of convexity in function composition [14]. We further observe that the objective function (11) is a convex function over $P_{st,in}(t)$ for $t \in [1,T]$ due to the assumption that $\xi\left(t,\omega(t)\right)$ is fixed. Combined with the fact that Eqns. (12) and (13) are linear inequality constraints of $P_{st,in}(t)$'s, we have proved the validity of the observation. ∎

Based on the above observation, we adopt an effective heuristic method to derive a near-optimal solution of the storage control problem. It is an iterative method. In each iteration, the heuristic method will maintain an estimate of the energy price $\xi\left(t,\omega(t)\right)$ in each time slot t (the energy price is assumed to remain fixed in this iteration), and then perform convex optimization with polynomial time complexity to derive the optimal $P_{st,in}(t)$ for $t \in [1,T]$. At the end of each iteration, the estimate of energy price $\xi\left(t,\omega(t)\right)$ will be updated based on the new $\omega(t)$ profile derived from the optimal $P_{st,in}(t)$ for $t \in [1,T]$. The heuristic method will converge to an effective near-optimal solution of the original storage control problem. Details of the heuristic are provided below.

Algorithm *NBSC* (Λ_{task})
```
Calculate P_load(t) and Σ_{i=1}^{N} I_i(λ_i)
Loop until convergence
¦ Update ξ(t, ω(t)) for all slot t
¦ Run convex optimization solver to find the optimal P_st,in(t)
¦ Calculate the optimal P_st(t) by (4)
Return Λ_st
```

Fig. 5. Pseudo code for NBSC algorithm.

IV. EXPREIMENTAL RESULTS

In order to demonstrate the effectiveness of the proposed NBCM algorithm described in Section III.B, various test cases corresponding to the aforesaid system model are examined. As mentioned in Section II.A, we consider the cost minimization problem in one day and use $T = 24$ and $D = 60$ minutes in our experiments. Since D is set to one hour, the power consumptions of tasks and the storage control decisions are also determined with the granularity of one hour. For the residential user of interest, we use realistic conversion efficiency values in the user's power conversion circuits in the range of 85% to 95%, and we apply typical parameters of realistic residential energy storage systems. Furthermore, a realistic residential PV generation system is considered, and we use PV power profiles measured at Duffield, VA, in the year 2007, an example of which is in Figure 6 (a).

We assume that the user provides the duration, inconvenience cost, power profile, preferable earliest start time and deadline for each task at the beginning of the day, and the utility company also provides the TOU-dependent price as well as the instantaneous power consumption-dependent price at the beginning of the day. The TOU-dependent price part, i.e., the base energy price when the grid power consumption $\omega(t) = 0$, is shown in Figure 6 (b), and the power consumption-dependent price part is monotonically increasing with the user's real-time grid power consumption $\omega(t)$.

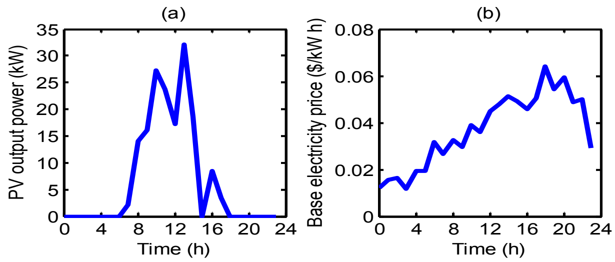

Fig. 6. (a) PV output power meansured in a day at Duffield, VA, (b) Base (TOU) electricity price provided by the utility company in a day.

We compare the total energy cost among the following (proposed and baseline) techniques:

- Greedy task scheduling (Greedy): where every task is greedily scheduled to minimize its own energy cost in one pass with the optimal storage control. This Greedy baseline is designed to demonstrate the effectiveness of the proposed negotiation-based task scheduling algorithm.
- Task scheduling without storage (No-Storage): where the NBTS algorithm is applied in an iterative manner for the user without energy storage system. The No-Storage baseline is used to show the significance of the energy storage.
- Ideal storage (Ideal-Storage): where most parts are the same with the proposed algorithm except that the storage control algorithm ignores the energy loss caused by the rate capacity effect. The Ideal-Storage baseline is presented to demonstrate the importance of a high accuracy storage model in storage control algorithm.
- The NBCM: where the NBTS and NBSC are implemented in an iterative manner as presented in Figure 3.

In the first experiment, we adopt a fixed storage system capacity of 24kWh and randomly generate test cases with the task number ranging from 5 to 50. The preferable time window of each task is randomly generated, and the power consumption of each task is also randomly generated with an expected value which is inversely proportional to the number of tasks (this is because we want to keep the total load energy consumption nearly a constant value). Table I reports the total energy cost among three baselines and the proposed NBCM algorithm, and the cost reduction with respect to different baselines are also presented. One can observe that the proposed NBCM algorithm consistently achieves lower energy cost compared with the three baseline algorithms, with the maximum energy cost reduction up to 64.22%. Among the three baselines, the Ideal-Storage baseline results in the highest performance because it is equipped with a storage system and utilizes the NBTS algorithm for task scheduling (which is closest to the proposed NBCM algorithm.)

In order to show the effect of the capacity of energy storage, in the second experiment we fix the task number $N = 50$, and compare the results of the three baselines and the proposed NBCM algorithm with the storage capacity in the range of 5kWh to 35kWh, as shown in Figure 7. The proposed NBCM algorithm achieves the lowest total energy cost consistently under different storage scenarios.

V. CONCLUSION

A formularization of joint task scheduling and energy storage control for residential energy consumers with PV and energy storage facilities was presented. An iterative negotiation-based cost minimization algorithm was proposed, which rips-up and reschedules all tasks under fixed storage control scheme and derives new charging/discharging scheme for the energy storage based on the latest task scheduling in each iteration. The concept of congestion was introduced to the proposed algorithm and a near-optimal storage control algorithm is implemented to solve convex optimization problem(s) with polynomial time complexity. The results were compared to three baseline methods and demonstrated significant cost savings under different scenarios.

TABLE I. PERFORMANCE OF THREE BASELINES AND NEGOCIATION-BASED COST MINIMIZATION ALGORITHM WITH DIFFERENT NUMBERS OF TASKS

Task Number	*Total energy cost*				*Cost reduction with respect to*		
	Greedy	*No-Storage*	*Ideal-Storage*	*NBCM*	*Greedy*	*No-Storage*	*Ideal-Storage*
5	23.42	16.21	10.43	8.38	64.22%	48.30%	19.65%
10	8.54	8.67	5.76	5.32	37.70%	38.64%	7.64%
15	4.41	5.29	3.01	2.87	34.92%	45.75%	4.65%
20	4.51	5.84	3.98	3.62	19.73%	38.01%	9.05%
25	8.55	9.02	7.84	6.24	27.02%	30.82%	20.41%
30	22.43	14.01	10.99	10.43	53.50%	25.55%	5.10%
35	15.28	11.03	9.50	8.09	47.05%	26.65%	14.84%
40	11.95	12.46	10.00	8.52	28.70%	31.62%	14.80%
45	5.82	8.55	5.86	5.32	8.59%	37.78%	9.22%
50	8.73	10.65	8.09	7.12	18.44%	33.15%	11.99%

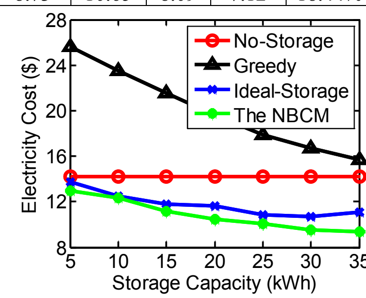

Fig. 7. Results of the three baselines and the NBCM using different storage capacities under a fixed task number.

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