# Analysis of Substrate Thermal Gradient Effects on Optimal Buffer Insertion

Amir H. Ajami and Massoud Pedram
University of Southern California

Kaustav Banerjee Stanford University

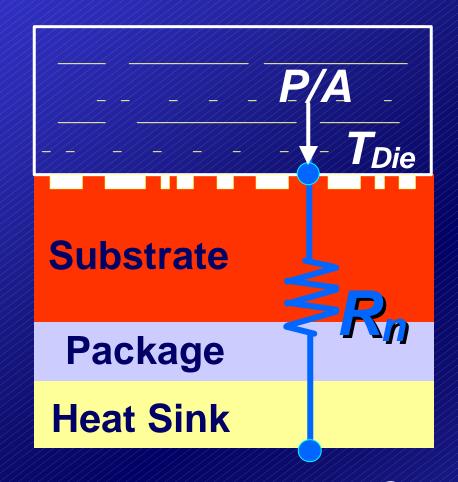




## **Outline**

- Introduction
- Non-Uniform Chip Temperature Profile
- Buffer Insertion Techniques
- Temperature-Dependent Buffer Insertion
- Summary

# **Average Chip Thermal Model**



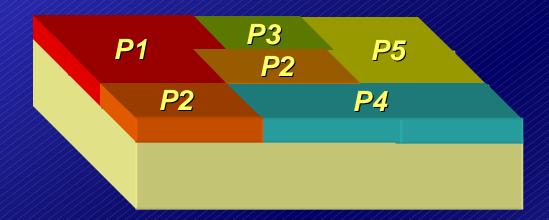
 $T_a = 25$  °C

1-D heat conduction model

$$T_{Die} = T_a + R_n \left(\frac{P}{A}\right)$$

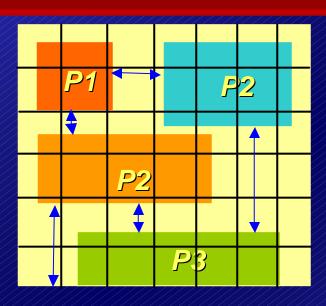
♣ Due to excessive Joule heating and the distance from the heat-sink, global interconnect lines are the hottest locations inside the chip

## Non-Uniform Substrate Power Map



- Substrate power generation distribution is generally non-uniform
  - Functional block clock gating
  - System-level power management
  - Non-uniform distribution of gate sizing and switching activities in different blocks

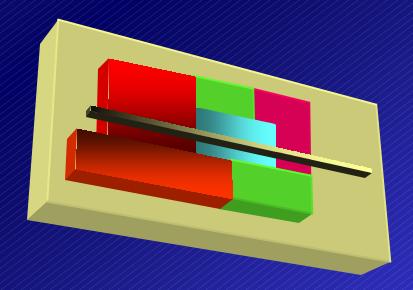
## **Substrate Temperature**



- Consider 4 neighbors of each square and solve 1-D heat conduction model
- Generating matrix  $\mathbb{R}_{t}$   $\mathbb{P} = \mathbb{T}$  using 6 neighbors of each grid in 3-D space
- Using FST to speed up the computation (Kang et al.)

# Non-Uniform Substrate Temperature

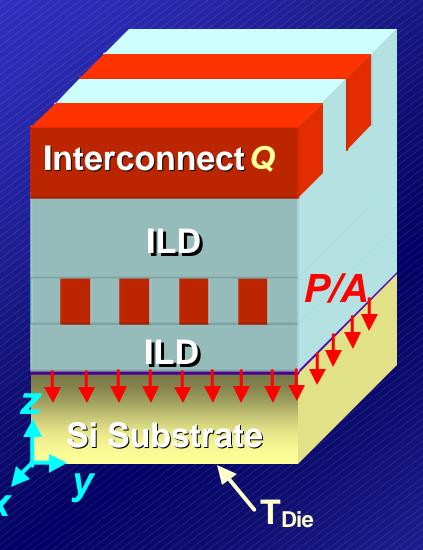
- Substrate thermal profile is non-uniform
  - \* Thermal time constant is of the order of ms
  - Switching activities in the block level are more important
  - Introduces non-uniformity in the global interconnect thermal profile



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## **Interconnect Thermal Profile**



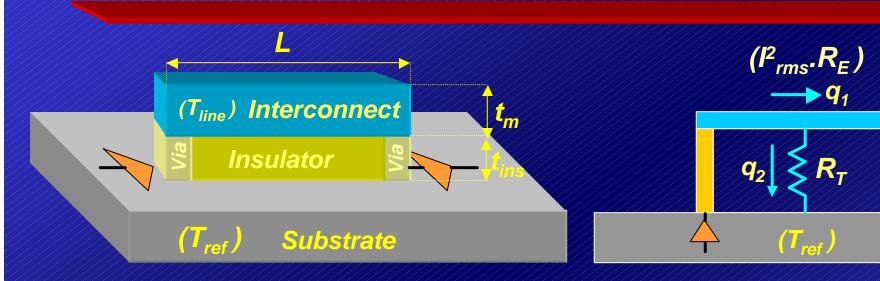
Three dimensional heat conduction in steady state

$$\tilde{N}^2 T = 0$$

♦ With an effective heat generation Q in the interconnect and a constant thermal conductivity k<sub>m</sub>

$$\nabla^2 T + \frac{Q}{k_m} = 0$$

## 1-D Heat Equation for Interconnects



$$\frac{d^2T_{line}}{dx^2} = -\frac{Q}{k_m}$$

$$\frac{d^2T_{line}(x)}{dx^2} = ?^2T_{line}(x) - ?^2T_{ref}(x) - ?$$

$$f(L, t_m, k_m, t_{ins}, k_{ins}, l_{rms}, R_E)$$

$$Q = q_1 - q_2$$

$$\lambda$$
 and  $\theta$  are constants  $f(L, t_m, k_m, t_{ins}, k_{ins}, I_{rms}, R_E)$ 

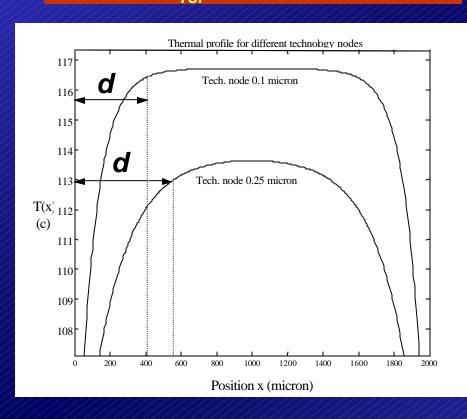
# **Spatial Temperature Distribution**

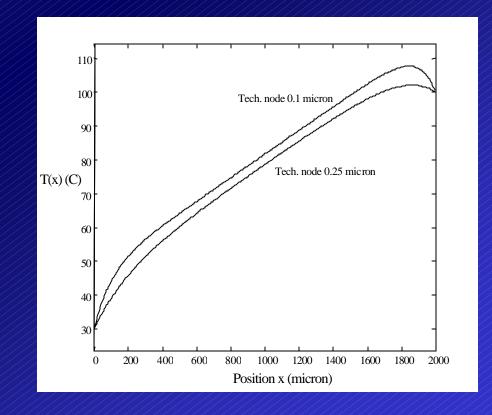
T(x=0) = 100 °CT(x=2000) = 100 °C

L=2000 mm

T(x=0) = 30 °CT(x=2000) = 100 °C

#### $T_{ref} = 100 \, ^{\circ}C$





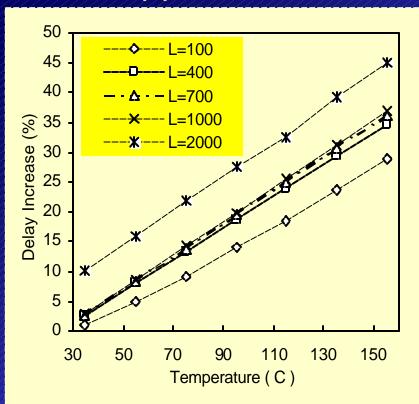
# Temperature Dependency of Delay

Interconnect delay dependent on T due to the T dependence of the resistance

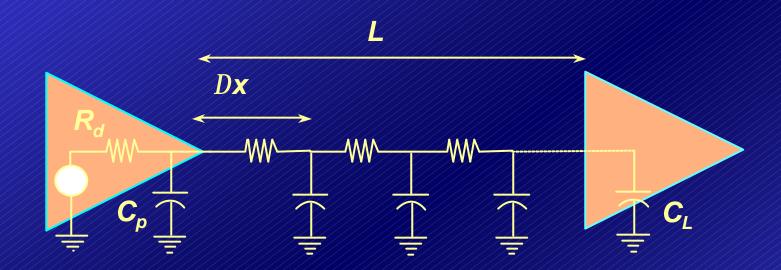
$$r(x) = ?_0(1 + BT(x))$$

- •r<sub>0</sub>: resistance per unit length at reference temperature
- •b: temperature coefficient of resistance (1/°C)





## Non-Uniform Temperature-Dependent Delay



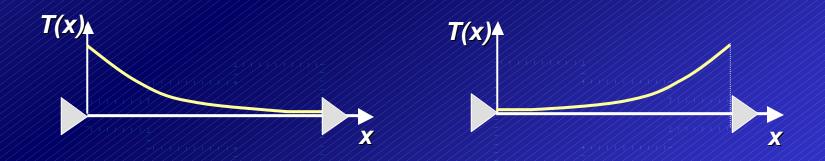
$$D = R_d (C_P + C_L + \int_0^L c_o(x) dx) + \int_0^L r_o(x) (\int_x^L c_o(h) dh + C_L) dx$$

$$D = D_o + (c_o L + C_L) ?_o R \int_0^L T(x) dx - c_o ?_o R \int_0^L x T(x) dx$$

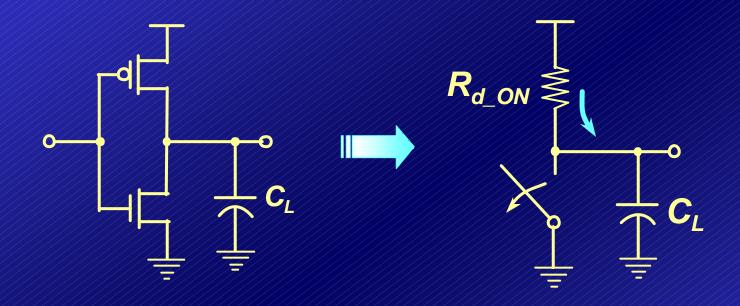
 $D_0$  is the Elmore delay model at reference temp.

## **Direction of Thermal Profiles**

- Decreasing (increasing) thermal profile is equivalent to increasing (decreasing) sizing profile for uniform resistance wire (DAC'01)
- Increasing thermal profile has better performance than that of decreasing thermal profile (optimal wire sizing)



# **Inverter ON-driving Resistance**



$$R_{d\_ON} \cong \frac{L_{eff}/W}{\mu C_{ox}(V_{DD} - V_{T})}$$

•  $V_T$  and m are dependent on the cell temperature

# Temperature-dependent R<sub>d</sub>

**Q**<sub>B</sub>: Depletion region charge

Cox: Gate oxide capacitance

E<sub>a</sub>: Energy gap of Silicon

g: Electron charge

**u**: Electron mobility

W: Gate width

**L**<sub>eff</sub>: Channel width

$$V_T \cong 2j_f - \frac{Q_B}{C_{ov}} \cong E_g - \frac{Q_B}{C_{ov}}$$

$$\frac{\partial V_T}{\partial T} = \frac{E_g/q + V_T}{T}$$

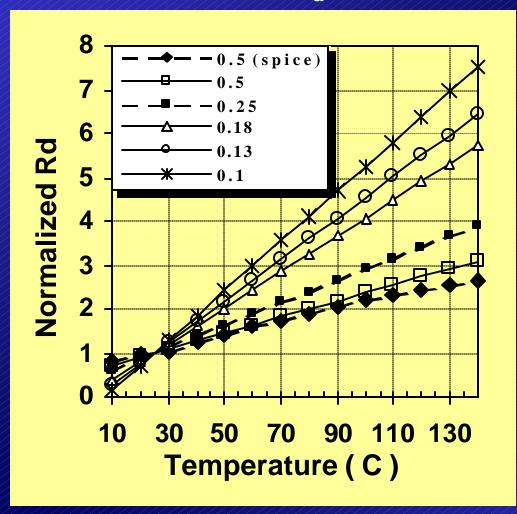


$$\frac{?R_d}{R_d} = \frac{E_g/q + V_T}{V_{DD} - V_T} \times \frac{?T}{T}$$

$$E_g/q \gg 1.12 \text{ V}$$

# ON-Resistance (R<sub>d</sub>) Variations

#### Normalized to R<sub>d</sub> at 25°C



0.25 
$$\mu$$
m  $V_T$ =0.6  $\forall$   $V_{dd}$ =3.3  $\forall$  0.18  $\mu$ m  $V_T$ =0.36  $\forall$   $V_{dd}$ =1.8  $\forall$  0.13  $\mu$ m  $V_T$ =0.3  $\forall$   $V_{dd}$ =1.5  $\forall$  0.10  $\mu$ m  $V_T$ =0.24  $\forall$   $V_{dd}$ =1.2 $\forall$ 

$$R_d(x) = R_{d0}(1 + B_cT(x))$$

♣ Thermal dependency of R<sub>d\_ON</sub> is much severe than that of R<sub>int</sub>

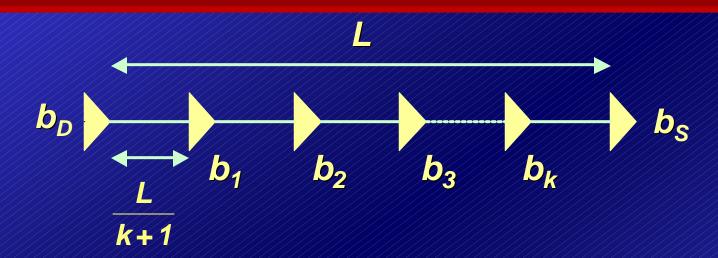
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## **Buffer Insertion**

- Improving the performance in signal nets with high capacitive loads by inserting buffers
- Finding the number of inserted buffers, their sizes and locations along the the net in order to minimize the delay
- In a given technology, the critical length between each two buffers and optimal buffer sizes can be extracted from the technology parameters (Otten et al., Alpert et al.)

## Methodology



$$I_{crit} = \sqrt{\frac{r_0 c_0 (1 + \frac{c_p}{c})}{rc}}$$

$$\mathbf{S}_{opt} = \sqrt{\frac{\mathbf{r}_{o}\mathbf{c}}{\mathbf{r}\mathbf{c}_{o}}}$$

r<sub>0</sub>: min. size transistor output resist.
 c<sub>0</sub>: min. size transistor input cap.
 c<sub>p</sub>: min. size transistor parasitic cap.
 r: unit length line resistance
 c: unit length line capacitance

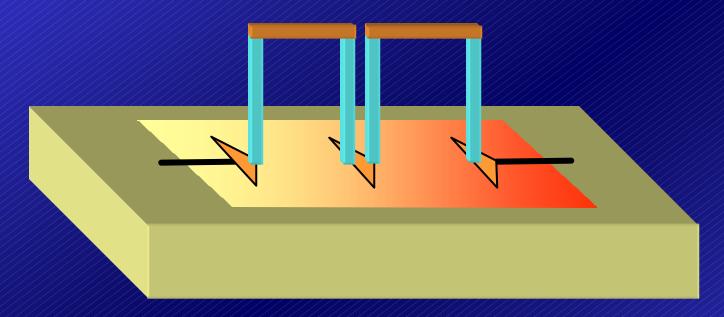
## Methodology

- Equal distances between each two adjacent buffers while having identical source and sink buffers
- Uniform line resistance per unit length r and min size driving resistance r<sub>o</sub>

$$k = \lfloor -0.5 + \sqrt{1 + \frac{2rcL^2}{R_d(C_L + C_p)}} \rfloor$$

$$R_d = \frac{r_0}{S_{opt}} \quad C_L = c_0.S_{opt} \quad C_P = c_p.S_{opt}$$

## Effects of Non-uniform Substrate Temp.

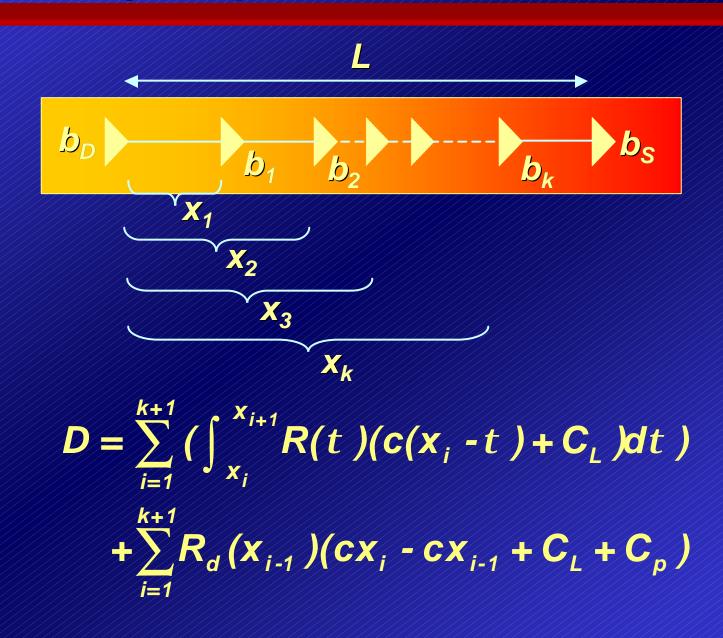


- Non-uniform substrate temperature causes:
  - Non-uniform interconnect resistance profile
  - Non-uniform ON-driving resistance profile for placed buffers

## **Outline**

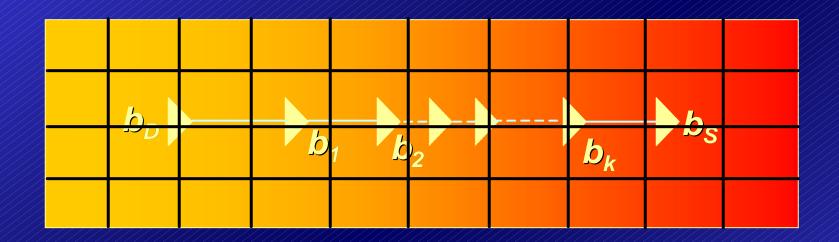
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# Thermally-Dependent Buffer Insertion



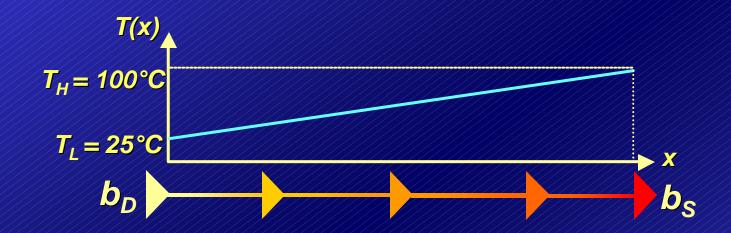
ICCAD'01

## **Assumption**



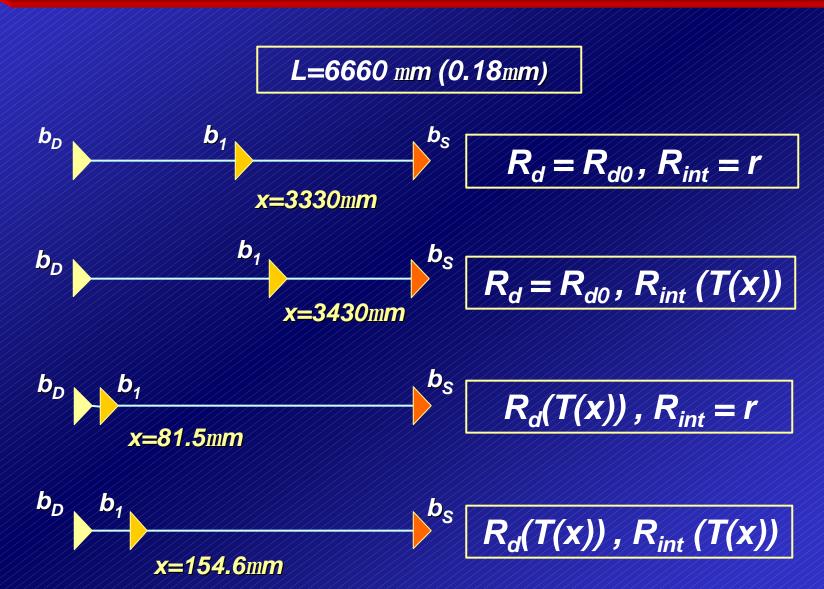
- Temperature of each grid square is a function of the total power consumption of gates located in that square
- In steady state, each inserted gate reaches to the temperature of its surrounding area

# R<sub>d</sub> vs. R<sub>int</sub> Thermal Dependencies



- Gradually increasing R<sub>int</sub> pushes the inserted buffers toward the sink buffer
- Gradually increasing R<sub>d</sub> pushes the buffers toward the source buffer

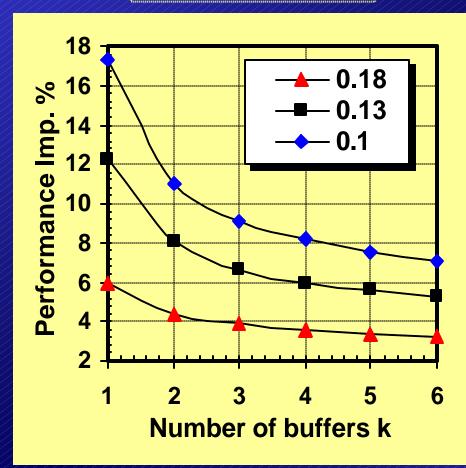
## **Buffer Movements**

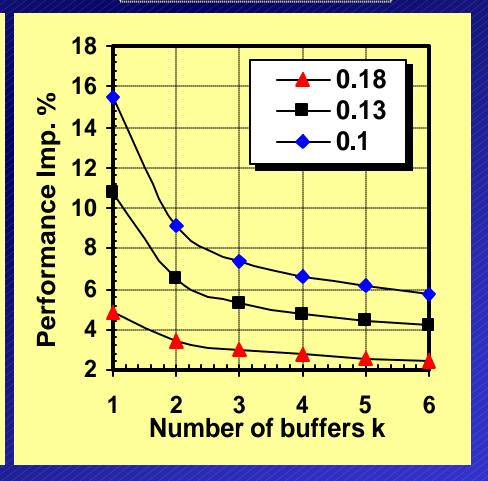


## Performance Improvement

 $R_d(T(x))$ , Rint = r

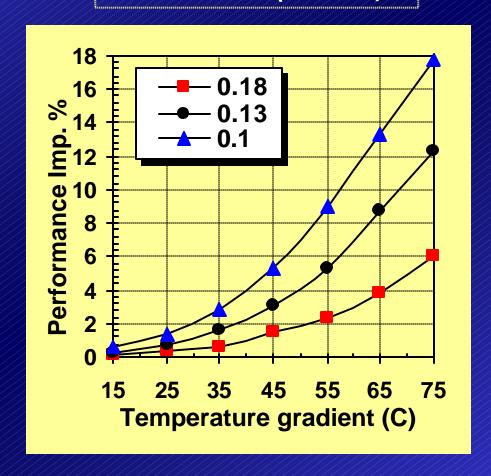
 $R_d(T(x))$ , Rint(T(x))





# **Effect of Thermal Gradient Magnitude**

L=6660 mm (0.18mm)



## Summary

- Due to different switching activities along with low power design policies, substrate & interconnect thermal maps are non-uniform
- Substrate thermal non-uniformities:
  - Affects the signal performance in interconnects
  - Severely impacts the device switching performance
  - Have serious effects on different EDA flow steps, specifically the buffer insertion routines
- Non-uniform substrate thermal profiles must be considered in the design flow of highperformance VLSI systems